Fight Cartels or Control Mergers? On the Optimal Allocation of Enforcement Efforts within Competition Policy

Andreea Cosnita-Langlais[*†] **and Jean-Philippe Tropeano**[‡]

May 19, 2010[§]

Abstract

This paper deals with the optimal enforcement of the competition law in terms of merger and anti-cartel policies. We examine the interaction between these two branches of the competition policy given the cost of resources available to the competition agency and taking into account the ensuing incentives for firms' behavior in terms of choice between cartels and mergers. We are thus able to infer the optimal allocation of enforcement efforts between controlling mergers and fighting cartels, and thereby conclude on their optimal competition policy mix. We show for instance that to the extent that firms may switch from cartel to merger depending on the current focus of the competition law enforcement, applying a stricter merger control only pays if the cartel fighting policy is not too expensive.

Keywords: competition law enforcement, antitrust, merger control, anti-cartel policy

JEL classification: L41, K21, D82

---

[*]Université Paris Ouest Nanterre La Défense and EconomiX - FRE CNRS 3257

[†]Corresponding author: acosnita@u-paris10.fr, +33(0)140977798, 200 Av.de la République, 92001 Nanterre, France.

[‡]Université de Paris 1-Paris School of Economics; tropeano@univ-paris1.fr

[§]We are endebted to Tomaso Duso, Joe Seldeslachts, Lars Sorgard and Kenneth Fjell for helpful remarks and suggestions. We equally benefitted from those of the participants to the ACLE 2009 meeting in Amsterdam, the EconomiX Lunch Seminar at Univ. Paris Ouest Nanterre, the Department of Economics Seminar at the NHH in Bergen, the EALE 2009 conference in Rome and the Economics Departement seminar at Univ Paris Sud. All remaining errors are ours.

1. Introduction

The scrutiny of horizontal agreements by Competition Authorities experienced recently increasing severity. The new European Commission Merger Regulation that tightens the clearing criteria as well as the new European leniency program that undermines cartel stability are clear examples. The effectiveness of a stricter merger control crucially depends on the type of cartel-fighting that can be afforded by the competition agency (CA henceforth). The main argument behind this intuition is the increasing evidence that mergers and cartels are substitutable strategies for industry firms contemplating market coordination, which swap one for another depending on the relative focus of the CA on either merger control or cartel fighting. On the one hand, when mergers become harder to accomplish, cartel activities are adopted instead, as Neumann (2001) argues for German industries such as cement, food processing, and machine building. On the other, whenever cartel formation is restricted, firms tend to turn to mergers instead. In the words of Mueller (1996), "The Sherman Act was passed in 1890 as a reaction to the expansion of size and power of large enterprises in the United States, i.e., the rise of the 'trusts'. Ironically, by prohibiting cartels it encouraged firms to combine to avoid the costs of unbridled competition that were expected to follow the dissolution of the cartels, and thus helped precipitate the first great merger wave at the turn of the century." The American example of the Sherman and Clayton Acts is later confirmed in the UK by the outcome of the Restrictive Trade Practices Act (1956), which triggered a merger wave by outlawing cartels (Bittlingmayer (1985), Symeonidis (2002)). Similarly, based on the analysis of duration for a sample of international cartels prosecuted in the 1990s, Evenett et al (2001) argue that joint ventures and mergers are among the different measures adopted by firms for survival, in cartel-prone industries where cartel formation is restricted[1]. The choice to resort to merger instead of cartel, or the reverse, depends on the current enforcement of the competition policy, to the extent that the latter modifies the relative profitability of these two options.

In this paper we develop a very simple framework to determine the optimum enforcement effort of both merger control and cartel fighting, depending on the cost of cartel detection, the social loss from market coordination between poorly efficient firms and the social gain from mergers leading to cost savings. For this purpose, we also model the firms' decision to either

[1] According to Symeonidis (2002), the 1956 Restrictive Practices Act in the UK, by banning cartels, led to an increase in competition which resulted in lower profits, which in its turn prompted a profit-restoring increase in market concentration through mergers.

join a price-fixing cartel or notify and undertake a horizontal merger. The relative profitability of the two options will depend on the probability for a cartel to be convicted, as well as on a merger's associated cost and its capacity to achieve cost savings.

Cartel fighting is imperfect in our model, not all cartels get punished, and the probability to convict a cartel will depend on the amount of resources allocated to this branch of the competition policy, knowing that resources are costly. The amount of resources used for fighting cartel will be used to model the severity of the fight against price agreements. The enforcement of merger control is also imperfect, at least to the extent that one considers the current policy setting, as we do. Explicitly, the *ex ante* assessment of horizontal mergers (without any *ex post* review) inevitably gives rise to both type of errors, i.e. clearing anti-competitive mergers and banning cost-efficient pro-competitive ones. This occurs because of the asymmetric information between the competition agency and the merging partners on the actual probability for a given notified merger to be a cost-efficient one. To account for this, we model a competition agency dealing with a population of merger projects that differ along two dimensions: the degree of cost savings that can be obtained through the merger, and the riskiness of the project itself (i.e. the probability to actually achieve the cost savings).

The main trade-off that we put forward in this framework is the following. Whenever more resources are invested into fighting cartels, not only will more cartels be detected and punished and thus welfare losses avoided (basically, a detection effect), but also firms will be prompted to abandon cartel formation and undertake horizontal mergers instead (a so-called selection effect). This may improve welfare as long as the newly-triggered mergers are efficient. However, given the imperfect merger control, it may also turn out to be a welfare-decreasing strategy. On the other hand, increasing the severity of merger control - which is not the same as its accuracy - means that the least socially-profitable merger projects will turn into price-fixing cartels, which will lower expected welfare. The opportunity to resort to a more or less lenient cartel-fighting policy and merger control will depend on the cost of available resources and on the level of welfare gains and losses that are likely to obtain from mergers.

Based on this, we are able to provide the following results. First of all, depending on the relative cost of resources allocated to fighting cartels, the public actions towards horizontal mergers and price agreements may be either substitutable or complementary. As long as an intense cartel detection is not affordable, due to a high cost of budget resources, we are able to show that a tougher merger control saves on these costly anti-cartel resources - hence the substitutability between the two. In turn, when the CA affords to intensify cartel fighting thanks to low-cost resources, we find that the soft merger policy saves on anti-cartel resources, leading to complementarity between merger control and cartel fighting.

In terms of welfare, we are able to identify the optimal competition policy mix between merger control and cartel fighting, according to the different levels of cost of anti-cartel resources. Given the above mentioned detection and selection effects, we obtain that for very expensive anti-cartel action, welfare is maximized by soft merger control coupled with intense cartel fighting. If the latter becomes less expensive, then the soft merger policy is still optimal, but together with lenient anti-cartel action. Finally, for very cheap anti-cartel resources, welfare is maximized by a combination of strict merger policy and very intense cartel fighting.

This is to our knowledge the first research paper to ask the question of the optimal competition law enforcement mix between merger control and cartel fighting. In a related but different context, a similar question was raised by Aubert and Pouyet (2004) concerning the relationship between cartel-fighting and sectorial regulation[2]. As far as antitrust and merger control go, the only contribution, albeit from a positive perspective, is that of Mehra (2008), which formalizes the firms' choice between merger and cartel depending on the private cost incurred on account of the current law enforcement (the fine in case the cartel is detected).

The rest of the paper is structured as follows. We first present the framework of our analysis, then provide the results starting with the private choice by industry firms between cartel and horizontal merger. Next we infer the optimal public choice in terms of cartel deterrence effort, and conclude on the optimal competition policy mix between cartel fighting and merger control. The paper closes with some ideas for enriching the basic framework retained and thus extend our results. All formal proofs are grouped in a final technical appendix.

2. Model

2.1. Framework

The analysis will involve two risk-neutral agents, a group of industry firms on the one hand and the competition agency on the other. The former choose between either horizontal merger

[2] See also Bensaid et al. (1995), which investigate the optimality of having a unique antitrust authority to deal with both cartel and mergers, or whether it is on the contrary best to separate the two on account of information and incentives issues.

or cartel, whereas the latter decides on the amount of resources dedicated to fighting cartels, as well as which mergers are to be cleared or banned.

Consider for this purpose the following reduced-form setting, in which the industry firms may coordinate in order to improve profitability either by colluding or by engaging in a horizontal merger[3]. If the cartel is not detected and punished, it provides a joint collusive payoff of π. It may however be punished with probability $a(\Delta)$, $a(0) = 0$, where $\Delta \in \left[0, \overline{\Delta}\right]$ stands for the amount of resources spent by the CA on fighting cartels. If so, then the ensuing payoff for the firms will be the status-quo competition joint revenue π^C, where $\pi^C < \pi$. We assume that the total cost incurred by the CA to fight against collusive agreements is $k\Delta$, where k is the constant marginal opportunity cost of funds used to fight cartels. Also, the punishment technology is linearly increasing with the amount of available resources: $a'(\Delta) > 0$ and $a''(\Delta) = 0$, with $a(0) = 0$ and $a(\overline{\Delta}) = 1$. Basically, the cost and the effectiveness of the anti-cartel fighting (represented by $a(\Delta)$) exhibit constant returns to scale.

The horizontal merger on the other hand is not only a legal means to achieve coordination, but also a source of cost savings. These efficiency gains, denoted e, will be achieved with probability ρ, leading to a joint profit of π_e. However, with the complementary probability the merger will fail to do so, leading to a joint profit of π. The probability ρ is distributed on $[0, 1]$ according to a uniform cumulative distribution function $G(\rho)$ and density $g(\rho)$. For simplicity, we assume that there are only two types of cost savings, either high or low, $\overline{e} > \underline{e}$, both occurring with equal probability. Joint profit will be of course increasing with the amount of these cost-savings, with $\pi_{\overline{e}} > \pi_{\underline{e}} > \pi$. To simplify notations, let $\pi_{\overline{e}} = \overline{\pi}$ and $\pi_{\underline{e}} = \underline{\pi}$. We assume that this efficiency gains parameter e is observable, whereas the probability of achieving the cost savings is not. In other words, only firms know the precise value of ρ, the CA only knows its distribution[4]. Finally, industry firms incur a fixed cost K in order to merge. This assumption summarizes the fact that coordination through merger is likely to be costlier than through collusion, to the extent that the price-fixing does not require a structural change in the organization of the partners, but also the fact that there are legal constraints (notification and lawyers' fees) to be obeyed and legal costs to be incurred for a merger to be started, whatever

[3] Our results do not depend on the type of competition (price or quantities) prevailing on the market.

[4] In practice, merging parties typically provide to the CAs estimations of the cost savings generated by their merger. However, proving convincingly and beforehand to the CA that the cost savings will indeed materialize afterwards is altogether a different matter. Our framework basically focuses on this latter uncertainty.

its outcome[5].

In terms of competition policy, the CA maximizes the expected Consumers' Surplus from both fighting cartels and controlling mergers, net of the cost of resources. Let W denote consumers' welfare following a successful cartel, and W^C the status-quo competition welfare without any coordinated behavior, with $W^C > W$. Concerning the merger policy, let the post-merger consumers' welfare be either W_e or W, depending on whether the cost savings materialize or not. To simplify notations, let $W_{\overline{e}} = \overline{W}$ and $W_{\underline{e}} = \underline{W}$. Furthermore we assume that $\overline{W} > \underline{W}$, meaning that the more efficiency gains, the higher the consumers' surplus. In practice, mergers get cleared or banned depending on the expected competitive impact, which is basically assessed in terms of expected post-merger price variation. Note however that due to the imperfect information on the probability for the cost savings to actually occur, and the firms' possible choice of cartel instead of merger, merger control may end up clearing welfare-lowering mergers.

The timing of the game will be the following:

At the first stage the CA chooses the amount of resources Δ available for fighting against cartels, and also the severity of the merger control policy it will apply. Explicitly, it sets the threshold $\widehat{e}, \widehat{e} \in \{\overline{e}, \underline{e}\}$ such that it will only clear mergers with $e \geq \widehat{e}$. Two cases may occur. According to the "soft" merger policy all mergers will actually be cleared, because $e \geq \widehat{e} = \underline{e}$, for any e. Under the "strict" merger policy, only the high-efficiency mergers get cleared, as implied by $\widehat{e} = \overline{e}$.

At the second stage, industry firms make their coordination choice between horizontal merger and collusive behavior. If merger is chosen, they notify it to the CA.

At the final stage, notified mergers are cleared or banned according to the threshold set at the first stage. If there was no notification, then the cartel is convicted with probability $a(\Delta)$ and the market is forced back to its status-quo competition situation. Otherwise, the industry ends up with the collusive market outcome.

The relevant equilibrium concept is the subgame perfect Nash equilibrium, and in what follows we solve the game backwards.

[5] Basically, we normalize to zero the private costs of undertaking cartels (establishing productions quotas, monitoring members, punishing deviations and so on). Thus K represents the relative cost of mergers w.r.t. cartels. We wish to stress that mergers require fixed and sunk costs rather than recoverable or variable ones.

2.2. The private choice between merger and cartel

When deciding how best to achieve profitable coordination, industry firms anticipate the outcome of the CA's decision at the final stage of the game. That means that the choice between horizontal merger and cartel is determined by the probability for a cartel to be punished on the one hand, and the type of merger control, more or less severe, on the other. The resulting expected profit will therefore lead to a self-selection effect according to which the merger will be chosen only if the probability to achieve cost savings is sufficiently high. In the Appendix we show that:

Lemma 1. *For any $e \in \{\overline{e}, \underline{e}\}$, there exists a probability threshold $\widehat{\rho}_e(\Delta, \widehat{e})$ decreasing with Δ, such that the merger is notified iff $\rho \geq \widehat{\rho}_e(\Delta, \widehat{e})$.*

The intuition is straightforward: merger is possibly costlier than cartel as a means to increase joint profits, so it takes a high enough probability of achieving the cost savings for the industry firms to prefer it to collusion. In addition, the higher the resources allocated to fighting cartels, and thus the higher the punishment probability in case of cartel, the higher the incentives for firms to prefer the merger instead. Finally, for a given amount of anti-cartel resources, there will be relatively more highly efficient mergers taking place rather than low efficient ones, because the relevant probability threshold is lower ($\widehat{\rho}_{\underline{e}}(\Delta, \widehat{e}) > \widehat{\rho}_{\overline{e}}(\Delta, \widehat{e})$). Basically, the higher the efficiency gains, the likelier the firms are to choose merger instead of cartel.

The level of K the merger fixed cost is naturally crucial for the trade-off to occur or not between merger and cartel from the firms' point of view. Otherwise, either cartel or merger is a dominant strategy for industry firms. For this reason we actually restrict the analysis to the relevant range for K, defined in the Appendix.

2.3. The public trade-off: anti-cartel resources vs. severity of merger control

Going back to the previous stage, we discuss in this section the outcome of the CA's simultaneous choice of how many resources to allocate to fighting cartels and which mergers to ban. The CA anticipates the self-selection effect highlighted by Proposition 1, so it knows that its choice of a merger efficiency threshold and amount of anti-cartel resources will impact on the populations of possible cartels and mergers.

Taking into account this double choice, as well as the expected net consumers' welfare maximization objective and the cost of cartel-fighting resources, the general programme for the CA writes as follows:

$$\max_{\substack{\Delta\in[0,\overline{\Delta}]\\ \widehat{e}\in\{\underline{e},\overline{e}\}}} \sum_e \frac{1}{2}\left\{G\left(\widehat{\rho}_e(\Delta,\widehat{e})\right)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W\right]+\int_{\widehat{\rho}_e(\Delta,\widehat{e})}^{1}\left[\rho W_e+(1-\rho)W\right]g(\rho)d\rho\right\}-k\Delta$$

This expression summarizes the following trade-off on behalf of the CA.

The CA's strategy to fight cartels will deter part of the population of possible mergers from colluding, and from lemma 1 we know that it is the merger projects most likely to achieve the cost savings that will actually be notified. At the same time, the CA's choice of a merger control threshold implies that some merger projects may never get approved. Explicitly, given the binary setting for the cost savings parameter in our setting, the CA may allow either all mergers or only the highly efficient ones. The latter case corresponds as before mentioned to a stricter merger control which, although improving on average the outcome of merger control, will push the less efficient merger projects to become cartels instead, and hence may lower the final welfare outcome of the competition policy. The CA will solve this trade-off between the severity of merger control and the intensity of effort put into fighting cartels by identifying the optimal combination $(e,\Delta(e))$ in terms of merger threshold and amount of anti-cartel resources.

We start by examining the optimal amount of ressources allocated to cartel fighting for a given threshold $\widehat{e}$ that governs merger control.

2.4. The optimal choice of anti-cartel resources

We characterize the optimal level of ressources allocated to cartel fighting and we prove the following intermediary result:

Lemma 2. *For any type of merger control policy $\widehat{e}\in\{\underline{e},\overline{e}\}$, the optimal amount of resources to be allocated to fighting cartels, denoted by $\Delta^*(\widehat{e})$, is such that:*

$$\sum_{e\in\{\underline{e},\overline{e}\}}\frac{1}{2}\left\{\begin{array}{c}\underbrace{G\left(\widehat{\rho}_e(\Delta,\widehat{e})\right)\cdot\left[a'(\Delta)(W^C-W)\right]}_{\text{detection effect}}\\ \underbrace{-\widehat{\rho}_e'(\Delta,\widehat{e})\left[\widehat{\rho}_e(\Delta,\widehat{e})W_e+(1-\widehat{\rho}_e(\Delta,\widehat{e}))W-a(\Delta)W^C-(1-a(\Delta))W\right]}_{\text{Selection effect}}\end{array}\right\}=k$$

This means that at the optimum, whatever the type of merger control, the marginal welfare gain from increasing the amount of resources for fighting cartels needs to equal the marginal cost of this increase in resources.

The marginal benefit from enhancing cartel fighting can be decomposed into two effects. On the one hand, the more money is spent to convict cartels, more cartels get punished and thus more welfare losses are avoided. This is what we call a detection effect, which trivially improves expected welfare whatever the type of merger policy. On the other hand, there is also a possibly ambiguous selection effect, according to which some cartels being now deterred become mergers. This explains why in general $\Delta^*(\underline{e}) \neq \Delta^*(\overline{e})$. Indeed, the size of this selection effect depends on the type of merger control. With a soft merger policy, it concerns both possible types of merger, $\underline{e}$ and $\overline{e}$, but with the strict merger control, only the $\overline{e}$-types will possibly react in this way, because the $\underline{e}$-ones will not be allowed to merge, however large the incentives provided by the stronger anti-cartel action. Moreover, it can take both signs, to the extent that the deterred cartels are not necessarily pro-competitive mergers, i.e. they do not necessarily achieve the cost savings often enough to actually improve expected welfare on average.

Next we investigate the outcome of a change in merger policy (from soft to strict) in case the amount of resources allotted to cartel-fighting goes up:

Lemma 3. *The marginal welfare gain from enhancing the severity of the merger policy increases with the amount of resources allocated to fighting collusion:* $\frac{\partial}{\partial \Delta}(W(\overline{e}, \Delta) - W(\underline{e}, \Delta)) > 0$.

Lemma 3 basically presents a single-crossing condition, whose intuition builds on the selection effect before mentioned.

To start with, under the soft merger control this effect concerns both $\underline{e}$ and $\overline{e}$ types, whereas the stricter merger control restricts the number of acceptable mergers to the only $\overline{e}$. Therefore, when little money is put into fighting cartels, the soft merger policy allows a higher marginal welfare gain w.r.t. the strict one because and as long as the newly-triggered mergers are on average welfare-improving. However, as cartel fighting gets tougher and tougher through higher resources spending, the less likely the newly triggered mergers to achieve the cost savings, simply because the opportunity cost to undertake a merger is lower. This follows from Lemma 1. Consequently, the more resources are spent on fighting cartels, the lower the marginal social benefit of controlling more mergers trough the soft merger policy. Equivalently, the selection effect gradually diminishes and eventually turns negative. Conversely, the higher the gain to restrict the merger notifications to the only $\overline{e}$ types. On the whole, the marginal benefit from enhancing the severity of merger control (i.e. giving up on $\underline{e}$-mergers) increases with the amount of resources used to fight cartels.

Based on these two intermediary results, we have the following:

Proposition 1. *There exists a marginal cost of resources threshold $\widetilde{k}$ such that:*

(i) for $k \geq \widetilde{k}$, a stricter merger control leads to lenient cartel fighting, i.e. $\Delta^*(\underline{e}) > \Delta^*(\overline{e})$

(ii) for $k < \widetilde{k}$, a stricter merger control leads to intense cartel fighting, i.e. $\Delta^*(\underline{e}) < \Delta^*(\overline{e})$.

Proposition 1 basically concludes on the relationship between the optimal intensity of cartel fighting and the severity of merger control that is applied. It states that for relatively high costs of anti-cartel resources, cartel fighting and merger control are substitutable, whereas for relatively low costs, the two branches of the competition policy are complementary.

Proposition 1 orders actually the optimum levels of anti-cartel resources, one for each type of merger policy, depending on the marginal cost of these resources. It establishes two regimes, each corresponding to the relatively high or low cost of resources. The difference between the two regimes, which explains the change in the ranking of the two Δ^*, follows from Lemma 3, according to which the stricter the merger control, the higher the benefit from intensifying cartel fighting. The latter naturally depends on the cost of resources used.

To start with, for a high marginal cost of anti-cartel fighting, the optimal amount of resources used for this purpose will be low. In this case, the marginal social gain from intensifying anti-cartel action is higher under the soft merger policy, following Lemma 3. Equivalently, for the same marginal cost of anti-cartel resources, it takes a higher amount of resources to optimally fight against cartels under the soft merger policy. In other words, whenever cartel deterrence is quite expensive, the strict merger policy saves on the optimal amount of anti-cartel resources, which means that optimal cartel fighting and merger control enforcement are substitutes.

The intuition stems from the above-mentioned detection and selection effects. Any increase in the amount of resources used against cartels always yields a positive detection effect (more cartels are punished, and more welfare losses are avoided). But with a high cost of resources used, the so-called selection effect is also positive. Indeed, the high unit cost of resources affords few such resources, which implies that there will be relatively few mergers notified but with a high probability of cost savings, and relatively many cartels, under both merger policies (Lemma 1). Any supplementary euro invested into anti-cartel fighting will yield a positive selection effect, because it will trigger some more mergers, which are highly likely to achieve the cost savings and are therefore welfare-improving. Finally, this positive selection effect is enhanced under the soft merger policy, because it concerns both types of mergers, whereas the strict merger control restricts this effect to the only $\bar{e}$-type. To put it differently, for a high cost of resources used, investing one more euros into fighting cartels improves more welfare under the soft merger policy. Given that the marginal benefit from this under either merger policy is decreasing, it takes more resources for a given unit cost to equal it with the higher marginal social gain under the soft merger policy, or less such resources under the strict one, which in turn yields a lower marginal social gain. Hence the substitutability between the enforcement of the two branches of competition policy.

As the unit cost of resources goes down, the selection effect will also go down, and even become negative, unlike the detection effect, which is always positive. Indeed, the lower the unit cost of anti-cartel resources, the more of them are available, so more mergers get notified instead of cartels. But the social marginal benefit from having more mergers submitted under the soft merger policy diminishes and eventually turns negative, because the newly-triggered mergers are less and less likely to achieve the cost savings and thereby to improve welfare. Moreover, the strict merger control prevents this negative selection effect associated with the $\underline{e}$-mergers because it blocks them altogether. On this account, Lemma 3 concluded that the more money spent on fighting cartels, the higher the marginal benefit from enhancing the severity of merger control.

Consequently, for a low unit cost of anti-cartel resources, making available many such resources, the same slight increase in their amount will yield a higher welfare gain under the strict merger policy. This means that for the same marginal cost of anti-cartel resources, it takes more money to optimally fight cartels under the strict merger control, i.e. to equalize the higher marginal gain of these resources with their marginal cost. By the same token, the same marginal welfare increase may be achieved under the soft merger regime with a lower anti-cartel effort. To sum up, whenever fighting cartels is relatively cheap, we find that the enforcement of merger control and the optimal anti-cartel effort are complementary.

2.5. Optimal competition policy mix

At the first stage of the game the CA makes the choice of the optimal competition policy mix in terms of merger control and cartel fighting. Following Proposition 1, if the cost of anti-cartel resources is rather high, the CA will choose between either the soft merger control combined with tough cartel deterrence or the strict merger policy combined with a more lenient anti-cartel action. On the contrary, if the cost of anti-cartel resources is rather low, the CA will balance the combination of soft merger control with lenient cartel fighting against that of strict merger control with tough cartel action. Below we provide the outcome of this trade-off:

Proposition 2. *There exists $k^* \in \left(0, \widetilde{k}\right)$ such that the optimum competition policy is the following:*

(i) strict merger control and intense cartel fighting for $k < k^*$

(ii) soft merger control and lenient cartel fighting for $k^* \leq k < \widetilde{k}$

(iii) soft merger control and intense cartel fighting for $\widetilde{k} \leq k$.

Starting with the last case, let us give the intuition for this policy-making result.

When cartel fighting is very costly, there is only a limited amount of resources to be spent for it. This implies in return that the relevant probability thresholds for mergers to be preferred are quite high, so basically very few mergers, of either $\underline{e}$ or $\overline{e}$ type, will actually be submitted. However, they will all be very likely to achieve the cost savings, and there are more of them submitted under the soft merger policy. In short, with a high marginal cost of anti-cartel resources, the before mentioned selection effect is positive under both merger policies, and enhanced enhanced under the soft one. And following Proposition 2, merger control and optimal cartel fighting are substitutable over the range $k \geq \widetilde{k}$. Moreover, since the detection effect is always positive, thus providing incentives for the CA to always apply the toughest possible anti-cartel action (i.e. the highest amount of resources available given their cost), this identifies the combination of soft merger control with tough cartel fighting as the optimal policy mix.

In turn, for a low cost of anti-cartel resources, the two branches of the competition policy are complementary, as indicates Proposition 1. Given the large amount of anti-cartel resources available thanks to their low marginal cost, the choice of industry firms will be biased towards mergers under both merger policies (the relevant efficiency probability thresholds are very low, so a lot of either type of mergers tend to be submitted). However, this also means that many of them will likely reduce welfare, because the latest among them are unlikely to achieve the cost savings. This is particularly true under the soft merger policy, which allows the $\underline{e}$-mergers to take place. Basically, for $k < k^*$ the selection effect on $\underline{e}$-mergers is negative and large. This welfare loss may be prevented however of the strict merger control were applied. The trade-off over the severity of merger control is unambiguously solved in favour of the strict merger policy when the opportunity cost of giving up on $\underline{e}$-mergers is low, i.e. the social benefit of such merger is low. Thus, for $k < k^*$, the complementarity between merger control and optimal cartel fighting comes down to a self-enforcing relationship: the strict merger control is better because it prevents the negative selection effect on $\underline{e}$-mergers. However, it also leaves no choice to these $\underline{e}$-mergers but to collude, so it takes an intense cartel fighting to optimally cope with them.

Finally, Proposition 2 identifies a medium range $\widetilde{k} > k \geq k^*$ for the unit cost of anti-cartel resources. In this case, merger control and cartel are still complementary, as stated by Proposition 1. So the CA has basically the choice between the soft merger control with lenient cartel fighting and the strict merger policy with intense cartel fighting. But should the strict merger control be applied, it would forbid all $\underline{e}$-mergers. And for this to be better in terms of welfare, an intense cartel fighting would be required, so as to deal with all the $\underline{e}$-types which would consequently undertake cartel. Yet, the unit cost of resources within this medium range is still too high to afford enough money for a sufficiently tough anti-cartel action. Equivalently, the welfare loss from giving up on $\underline{e}$-mergers would not be covered by the welfare gain from the stronger cartel fighting, given the cost of the latter. In other words, the opportunity cost of giving up on $\underline{e}$-mergers through the strict merger policy is still too high, which means that the so-called selection effect of $\underline{e}$-mergers is still positive, and thus playing against the strict merger control, even though this soft merger policy leads to clear some anticompetitive mergers. This explains why the combination of soft merger policy and lenient cartel fighting is optimal over this medium range of cost of resources.

3. Concluding remarks

This paper focuses on the optimal enforcement of the competition law in terms of merger control and anti-cartel policies. The observation of real-life market behavior indicates that often firms switch from cartels to mergers, or the reverse, depending on the current focus of competition agencies. To put it short, when fighting cartels became the top-ranking objective, markets have been known experience a surge in merger activity. Starting from this, we examined the interaction between the enforcement of the two branches of the competition policy, the merger control and the anti-cartel fighting. In so doing, we accounted for the resulting incentives for firms, which play an essential part in the final welfare outcome of the competition law enforcement. Given the cost of resources available for fighting cartels, we infer the optimal competition policy mix between controlling mergers and fighting cartels. We show for instance that applying a stricter merger control only pays in terms of welfare if the anti-cartel effort is not too expensive. The results would nevertheless benefit from enriching the current simple framework. Several alternative assumptions could be explored. Introducing a public cost for controlling mergers, or equivalently a total amount of resources available for both fighting cartel and controlling mergers, would enable a true resource-allocation analysis for these branches of the competition policy. On a more general level, the case of complementarity between the strategies of horizontal merger and cartel on behalf of industry firms is equally worth integrating into the framework. Such extension as well as several other alternative technical assumptions on the shape of cost functions and probability distributions are left for future research.

REFERENCES

Aubert, C. and J. Pouyet (2004) "Competition policy, regulation and the institutional design of industry supervision", Louvain Economic Review 70(2), p. 153-168

Bensaid, B., D. Encaoua and A. Perrot (1995) "Separating the Regulators to Reduce Risks Due to Overlapping Control", cahiers Eco&maths 95.36 - Université de Paris I Panthéon - Sorbonne

Bittlingmayer, G. (1985) "Did Antitrust Policy Cause the Great Merger Wave?", Journal of Law and Economics, Vol. 28, No. 1, p. 77-118

Evenett, S.J, M. C. Levenstein and V.Y. Suslow (2001) “International Cartel Enforcement: Lessons from the 1990s”, The World Economy, Vol. 24, N. 9, p. 1221-1245

Mehra, P. (2008) "Choice between Cartels and Horizontal Mergers", mimeo

Mueller, D.C. (1996) "Antimerger Policy in the United States: History and Lessons", Empirica 23, p. 229-253

Neumann, M (2001), Competition Policy: History, Theory and Practice, Edward Elgar

Symeonidis, G. (2002), The Effects of Competition: Cartel Policy and the Evolution of Strategy and Structure in British Industry, Cambridge, MA, MIT Press

4. Appendix

Proof of Lemma 1.

Horizontal merger is preferred to cartel as long as $\rho\pi_e + (1-\rho)\pi - K \geq a(\Delta)\pi^C + (1-a(\Delta))\pi$ $\Leftrightarrow \rho \geq \frac{a(\Delta)(\pi^C-\pi)+K}{\pi_e-\pi} = \widehat{\rho}_e(\Delta,\widehat{e})$. In addition, $\frac{\partial}{\partial\Delta}\widehat{\rho}_e(\Delta,\widehat{e}) = a'(\Delta)\frac{\pi^C-\pi}{\pi_e-\pi} < 0$, and also $\widehat{\rho}_{\overline{e}}(\Delta) < \widehat{\rho}_{\underline{e}}(\Delta)$ due to $\pi_{\overline{e}} > \pi_{\underline{e}}$. ■

Proof - Define the relevant range for K.

$\widehat{\rho}_e(\Delta,\widehat{e}) \in [0,1] \Leftrightarrow 0 \leq \frac{a(\Delta)(\pi^C-\pi)+K}{\pi_e-\pi} \leq 1 \Leftrightarrow a(\Delta)\left(\pi-\pi^C\right) \leq K \leq (\pi_e-\pi)+a(\Delta)\left(\pi-\pi^C\right)$.

■

Proof of Lemma 2.

To prove this result we need first explicit the expected welfare depending on the possible values of $\widehat{e} \in \{\underline{e},\overline{e}\}$:

The expected welfare for the soft merger policy writes

$$W(\underline{e}) = \tfrac{1}{2}\left\{G\left(\widehat{\rho}_{\underline{e}}(\Delta)\right)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W\right]+\int_{\widehat{\rho}_{\underline{e}}(\Delta)}^{1}\left[\rho\underline{W}+(1-\rho)W\right]g(\rho)d\rho\right\}$$
$$+\tfrac{1}{2}\left\{G\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W\right]+\int_{\widehat{\rho}_{\overline{e}}(\Delta)}^{1}\left[\rho\overline{W}+(1-\rho)W\right]g(\rho)d\rho\right\}-k\Delta$$

whereas that for the strict merger policy writes

$$W(\overline{e}) = \tfrac{1}{2}\left[a(\Delta)W^C+(1-a(\Delta))W\right]$$
$$+\tfrac{1}{2}\left\{G\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W\right]+\int_{\widehat{\rho}_{\overline{e}}(\Delta)}^{1}\left[\rho\overline{W}+(1-\rho)W\right]g(\rho)d\rho\right\}-k\Delta$$

Denote $MB(e)$ the first derivative for the expected welfare. One has:

$$MB(\underline{e}) = \frac{1}{2}\left\{\begin{array}{l}\left[G\left(\widehat{\rho}_{\underline{e}}(\Delta)\right)+G\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\right]\cdot a'(\Delta)\cdot\left[W^C-W\right]+\\ g\left(\widehat{\rho}_{\underline{e}}(\Delta)\right)\cdot\widehat{\rho}_{\underline{e}}'(\Delta)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\underline{e}}(\Delta)\underline{W}-(1-\widehat{\rho}_{\underline{e}}(\Delta))W\right]+\\ g\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\cdot\widehat{\rho}_{\overline{e}}'(\Delta)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\overline{e}}(\Delta)\overline{W}-(1-\widehat{\rho}_{\overline{e}}(\Delta))W\right]\end{array}\right\}-k$$

and

$$MB(\overline{e}) = \frac{1}{2}\left\{\begin{array}{l}\left[1+G\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\right]\cdot a'(\Delta)\cdot\left[W^C-W\right]+\\ g\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\cdot\widehat{\rho}_{\overline{e}}'(\Delta)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\overline{e}}(\Delta)\overline{W}-(1-\widehat{\rho}_{\overline{e}}(\Delta))W\right]\end{array}\right\}-k$$

For each type of merger policy and for a given k (not too high, of course) an interior solution $\Delta^*(e)$ obtains from $MB(e)=0$.

The solution $\Delta^*(e)$ is unique in each case, thanks to the following (the strict monotonicity of the MB functions):

$$\tfrac{\partial}{\partial\Delta}MB(\underline{e}) = \tfrac{1}{2}\left\{\begin{array}{l}\left[g(\widehat{\rho}_{\underline{e}}(\Delta))\cdot\underbrace{\widehat{\rho}_{\underline{e}}'(\Delta)}_{<0}+g\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\cdot\underbrace{\widehat{\rho}_{\overline{e}}'(\Delta)}_{<0}\right]\cdot\underbrace{a'(\Delta)}_{>0}\cdot\underbrace{\left[W^C-W\right]}_{>0}+\\ g(\widehat{\rho}_{\underline{e}}(\Delta))\cdot\underbrace{\widehat{\rho}_{\underline{e}}'(\Delta)}_{<0}\cdot\underbrace{\left[a'(\Delta)(W^C-W)-\widehat{\rho}_{\underline{e}}'(\Delta)\left(\underline{W}-W\right)\right]}_{>0}+\\ g\left(\widehat{\rho}_{\overline{e}}(\Delta)\right)\cdot\underbrace{\widehat{\rho}_{\overline{e}}'(\Delta)}_{<0}\cdot\underbrace{\left[a'(\Delta)(W^C-W)-\widehat{\rho}_{\overline{e}}'(\Delta)\left(\overline{W}-W\right)\right]}_{>0}\end{array}\right\}<0$$

and $\frac{\partial}{\partial\Delta}MB(\overline{e}) = \frac{1}{2}\left\{\begin{array}{l} \left[g\left(\widehat{\rho}_{\overline{e}}\left(\Delta\right)\right)\cdot\underbrace{\widehat{\rho}'_{\overline{e}}\left(\Delta\right)}_{<0}\right]\cdot\underbrace{a'(\Delta)}_{>0}\cdot\underbrace{\left[W^C-W\right]}_{>0}+ \\ g\left(\widehat{\rho}_{\overline{e}}\left(\Delta\right)\right)\cdot\underbrace{\widehat{\rho}'_{\overline{e}}\left(\Delta\right)}_{<0}\cdot\underbrace{\left[a'(\Delta)(W^C-W)-\widehat{\rho}'_{\overline{e}}\left(\Delta\right)\left(\overline{W}-W\right)\right]}_{>0} \end{array}\right\} < 0.$

In order to show that $\Delta^*(e)$ is an interior solution, we only need to prove that $MB(e) < 0$ for $\Delta \to \overline{\Delta}$.

This is the case as long as $a(\Delta)W^C + (1-a(\Delta))W - \widehat{\rho}_e\left(\Delta\right)W_e - (1-\widehat{\rho}_e\left(\Delta\right))W < 0$ for very large Δ. This implies in turn $\frac{a(\Delta)}{\widehat{\rho}_e(\Delta)} < \frac{W_e-W}{W^C-W}$. Since the RHS term is > 1 thanks to our assumptions on the welfare levels, it is enough to show that $\frac{a(\Delta)}{\widehat{\rho}_e(\Delta)}$ may be ≤ 1 for this condition to hold. Since $\widehat{\rho}_e(\Delta, \widehat{e}) = \frac{a(\Delta)(\pi^C-\pi)+K}{\pi_e-\pi}$, this means we actually have to check the compatibility of $a(\Delta) \leq \frac{a(\Delta)(\pi^C-\pi)+K}{\pi_e-\pi}$ with the other parameter conditions required so far. In short, we need to check that $K \geq a(\Delta)\left(\pi_e - \pi^C\right)$ is compatible with $a(\Delta)\left(\pi - \pi^C\right) \leq K \leq (\pi_e - \pi) + a(\Delta)\left(\pi - \pi^C\right)$. This is straightforward as long as we further restrict the relevant range for the fixed cost of merger: $a(\Delta)\left(\pi_e - \pi^C\right) \leq K \leq (\pi_e - \pi) + a(\Delta)\left(\pi - \pi^C\right)$. ■

Proof of Lemma 3.

The marginal welfare gain from increasing the severity of the merger policy from $\underline{e}$ to $\overline{e}$ writes:

$$MB(\overline{e}) - MB(\underline{e}) = \frac{1}{2}\left\{\begin{array}{l} a'(\Delta)\cdot\left[W^C-W\right]\cdot\left[1-G\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right)\right] \\ -g\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right)\cdot\widehat{\rho}'_{\underline{e}}\left(\Delta\right)\cdot\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\underline{e}}\left(\Delta\right)\underline{W}-(1-\widehat{\rho}_{\underline{e}}\left(\Delta\right))W\right] \end{array}\right\}$$

Taking the first derivative on the marginal benefit differential yields:

$$\frac{\partial}{\partial\Delta}\left(MB(\overline{e}) - MB(\underline{e})\right) = \frac{1}{2}\left\{\begin{array}{l} \underbrace{a'(\Delta)}_{>0}\cdot\left[W^C-W\right]\cdot\underbrace{\left(-g\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right)\cdot\widehat{\rho}'_{\underline{e}}\left(\Delta\right)\right)}_{>0} \\ -g\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right)\cdot\underbrace{\widehat{\rho}'_{\underline{e}}\left(\Delta\right)}_{<0}\cdot\underbrace{\left[a'(\Delta)(W^C-W)-\widehat{\rho}'_{\underline{e}}\left(\Delta\right)\left(\underline{W}-W\right)\right]}_{>0} \end{array}\right\} > 0$$

q.e.d. ■

Proof of Proposition 1.

Define $\widetilde{\Delta}$ such that $MB(\overline{e}) - MB(\underline{e}) = 0$. Let then $\widetilde{k} = k(\widetilde{\Delta})$.

In order to show that $\widetilde{\Delta}$ is an interior solution, we evaluate $[MB(\overline{e}) - MB(\underline{e})]$ for $\Delta \to \overline{\Delta}$ and for $\Delta \to 0$:

- if $\Delta \to \overline{\Delta}$ then $a(\Delta) \to 1$ and $\widehat{\rho}_{\underline{e}}\left(\Delta\right) \to 0$, so $G\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right) \to 0$, which leaves $[MB(\overline{e}) - MB(\underline{e})]_{\Delta\to\overline{\Delta}} = \left(a'(\Delta) - g\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right)\cdot\widehat{\rho}'_{\underline{e}}\left(\Delta\right)\right)\left(W^C - W\right) > 0$
- if $\Delta \to 0$ then $a(\Delta) \to 0$, therefore $[MB(\overline{e}) - MB(\underline{e})]_{\Delta\to 0} = -g\left(\widehat{\rho}_{\underline{e}}\left(\Delta\right)\right)\cdot\widehat{\rho}_{\underline{e}}\left(\Delta\right)\left(W - \underline{W}\right) < 0$

Given $\frac{\partial}{\partial \Delta}\left(MB(\overline{e}) - MB(\underline{e})\right) > 0$ from Lemma 3, one has that $\widetilde{\Delta}$ such that $MB(\overline{e}) - MB(\underline{e}) = 0$ belongs to $\left(0, \overline{\Delta}\right)$. $\blacksquare$

Proof of Proposition 2.

Let $W(e, \Delta^*(e))$ denote the maximum net consumers' welfare function for any merger policy type, $e \in \{\underline{e}, \overline{e}\}$. Explicitly, this yields:

$$W(\underline{e}, \underline{\Delta}^*(\underline{e})) = \frac{1}{2}\left\{G\left(\widehat{\rho}_{\underline{e}}\left(\underline{\Delta}^*\right)\right) \cdot \left[a(\underline{\Delta}^*)W^C + (1-a(\underline{\Delta}^*))W\right] + \int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 \left[\rho \underline{W} + (1-\rho)W\right] g(\rho)d\rho\right\}$$
$$+\frac{1}{2}\left\{G\left(\widehat{\rho}_{\overline{e}}\left(\underline{\Delta}^*\right)\right) \cdot \left[a(\underline{\Delta}^*)W^C + (1-a(\underline{\Delta}^*))W\right] + \int_{\widehat{\rho}_{\overline{e}}(\underline{\Delta}^*)}^1 \left[\rho \overline{W} + (1-\rho)W\right] g(\rho)d\rho\right\} - k\underline{\Delta}^*$$

and $W(\overline{e}, \overline{\Delta}^*(\overline{e})) = \frac{1}{2}\left[a(\overline{\Delta}^*)W^C + (1-a(\overline{\Delta}^*))W\right]$

$$+\frac{1}{2}\left\{G\left(\widehat{\rho}_{\overline{e}}\left(\overline{\Delta}^*\right)\right) \cdot \left[a(\overline{\Delta}^*)W^C + (1-a(\overline{\Delta}^*))W\right] + \int_{\widehat{\rho}_{\overline{e}}(\overline{\Delta}^*)}^1 \left[\rho \overline{W} + (1-\rho)W\right] g(\rho)d\rho\right\} - k\overline{\Delta}^*$$

First of all, note that the maximum net consumers' welfare functions are both decreasing with the cost of anti-cartel resources: $\frac{\partial}{\partial k}W(\underline{e}, \underline{\Delta}^*) = -\underline{\Delta}^*$ and $\frac{\partial}{\partial k}W(\overline{e}, \overline{\Delta}^*) = -\overline{\Delta}^*$ thanks to the envelope theorem.

The optimal choice in terms of competition policy is based on the following differential:

$$W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\overline{e}, \overline{\Delta}^*(\overline{e})) =$$
$$\left(\begin{array}{c} \frac{1}{2}\left\{G\left(\widehat{\rho}_{\underline{e}}\left(\underline{\Delta}^*\right)\right) \cdot \left[a(\underline{\Delta}^*)W^C + (1-a(\underline{\Delta}^*))W\right] + \int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 \left[\rho \underline{W} + (1-\rho)W\right] g(\rho)d\rho\right\} \\ -\frac{1}{2}\left[a(\overline{\Delta}^*)W^C + (1-a(\overline{\Delta}^*))W\right] \end{array}\right)$$
$$+\left(\begin{array}{c} \frac{1}{2}\left\{G\left(\widehat{\rho}_{\overline{e}}\left(\underline{\Delta}^*\right)\right) \cdot \left[a(\underline{\Delta}^*)W^C + (1-a(\underline{\Delta}^*))W\right] + \int_{\widehat{\rho}_{\overline{e}}(\underline{\Delta}^*)}^1 \left[\rho \overline{W} + (1-\rho)W\right] g(\rho)d\rho\right\} \\ -\frac{1}{2}\left\{G\left(\widehat{\rho}_{\overline{e}}\left(\overline{\Delta}^*\right)\right) \cdot \left[a(\overline{\Delta}^*)W^C + (1-a(\overline{\Delta}^*))W\right] + \int_{\widehat{\rho}_{\overline{e}}(\overline{\Delta}^*)}^1 \left[\rho \underline{W} + (1-\rho)W\right] g(\rho)d\rho\right\} \end{array}\right) +$$
$$k(\overline{\Delta}^* - \underline{\Delta}^*)$$
$$= \frac{1}{2}\left(\begin{array}{l} G\left(\widehat{\rho}_{\underline{e}}\left(\underline{\Delta}^*\right)\right) \cdot \left(a(\underline{\Delta}^*) - a(\overline{\Delta}^*)\right) \cdot \left(W^C - W\right) \\ + \int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 \left[\rho \underline{W} + (1-\rho)W - a(\overline{\Delta}^*)W^C - (1-a(\overline{\Delta}^*))W\right] g(\rho)d\rho \end{array}\right)$$
$$+\frac{1}{2}\left(\begin{array}{l} \left(W^C - W\right) \cdot \left(a(\underline{\Delta}^*) - a(\overline{\Delta}^*)\right) \cdot \left(G\left(\widehat{\rho}_{\overline{e}}\left(\underline{\Delta}^*\right)\right) - G\left(\widehat{\rho}_{\overline{e}}\left(\overline{\Delta}^*\right)\right)\right) \\ + \int_{\widehat{\rho}_{\overline{e}}(\underline{\Delta}^*)}^{\widehat{\rho}_{\overline{e}}(\overline{\Delta}^*)} \left[\rho \overline{W} + (1-\rho)W\right] g(\rho)d\rho \end{array}\right) + k(\overline{\Delta}^* - \underline{\Delta}^*)$$

Next we establish the variation of this maximum welfare differential with the cost of anti-cartel resources: $\frac{\partial}{\partial k}\left[W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\overline{e}, \overline{\Delta}^*(\overline{e}))\right] = \overline{\Delta}^* - \underline{\Delta}^*$

According to the two regimes defined in Proposition 1, one has that:

- for $k < \widetilde{k}$, $\Delta^*(\underline{e}) < \Delta^*(\overline{e})$, thus $\frac{\partial}{\partial k}\left[W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\overline{e}, \overline{\Delta}^*(\overline{e}))\right] > 0$
- for $k \geq \widetilde{k}$, $\Delta^*(\underline{e}) > \Delta^*(\overline{e})$, therefore $\frac{\partial}{\partial k}\left[W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\overline{e}, \overline{\Delta}^*(\overline{e}))\right] < 0$.

Moreover, we can evaluate the maximum welfare differential in $k = \widetilde{k}$, and for $k \to 0$ and also for k so large that $\Delta^* \to 0$:

(i) for $k=\widetilde{k}$, by definition $MB(\overline{e})=MB(\underline{e})$, therefore $\Delta^*(\underline{e})=\Delta^*(\overline{e})$.Consequently,
$\left[W(\underline{e},\underline{\Delta}^*(\underline{e}))-W(\overline{e},\overline{\Delta}^*(\overline{e}))\right]_{\Delta^*(\underline{e})=\Delta^*(\overline{e})}$
$=\int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^{1}\left[\rho\underline{W}+(1-\rho)W-a(\overline{\Delta}^*)W^C-(1-a(\overline{\Delta}^*))W\right]g(\rho)d\rho>0$ because for $\widetilde{k}$,
$MB(\overline{e})-MB(\underline{e})=$

$$=\frac{1}{2}\left\{\begin{array}{l} a'(\Delta)\cdot\left[W^C-W\right]\cdot\left[1-G\left(\widehat{\rho}_{\underline{e}}(\Delta)\right)\right] \\ \underbrace{-g\left(\widehat{\rho}_{\underline{e}}(\Delta)\right)\cdot\widehat{\rho}'_{\underline{e}}(\Delta)}_{>0}\cdot\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\underline{e}}(\Delta)\underline{W}-(1-\widehat{\rho}_{\underline{e}}(\Delta))W\right]\end{array}\right\}=0$$

with $a'(\Delta)\cdot\left[W^C-W\right]\cdot\left[1-G\left(\widehat{\rho}_{\underline{e}}(\Delta)\right)\right]>0$,
therefore $\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\underline{e}}(\Delta)\underline{W}-(1-\widehat{\rho}_{\underline{e}}(\Delta))W\right]<0$.

(ii) for k so large that $\Delta^*\to 0,\forall e$, then $a(\Delta^*)\to 0$, meaning that under both merger policies, there is actually no fighting against cartels whatsoever. As a result, $W(\underline{e},\underline{\Delta}^*(\underline{e}))-W(\overline{e},\overline{\Delta}^*(\overline{e}))>0$, because the soft merger control allows the both types to merge (and these mergers are highly likely to achieve the efficiency gains, since $\widehat{\rho}_e(\Delta,\widehat{e})\to 1$), while the strict merger regime does not.

(iii) for $k\to 0$, both $\underline{\Delta}^*(\underline{e})$ and $\overline{\Delta}^*(\overline{e})$ tend to their maximum values, but still $\underline{\Delta}^*(\underline{e})<\overline{\Delta}^*(\overline{e})$ thanks to Proposition 1. Given the monotonicity of $a(\Delta)$ and $\widehat{\rho}_e(\Delta)$ with Δ, one has $a(\underline{\Delta}^*)<a(\overline{\Delta}^*)$ and $\widehat{\rho}_{\overline{e}}\left(\overline{\Delta}^*\right)<\widehat{\rho}_{\overline{e}}(\underline{\Delta}^*)$. This yields the following when evaluating the maximum welfare differential for or $k\to 0$:
$\left[W(\underline{e},\underline{\Delta}^*(\underline{e}))-W(\overline{e},\overline{\Delta}^*(\overline{e}))\right]_{k\to 0}=$

$$\frac{1}{2}\left(\begin{array}{l} G\left(\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)\right)\cdot\underbrace{\left(a(\underline{\Delta}^*)-a(\overline{\Delta}^*)\right)}_{<0}\cdot\left(W^C-W\right) \\ +\int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^{1}\left[\rho\underline{W}+(1-\rho)W-a(\overline{\Delta}^*)W^C-(1-a(\overline{\Delta}^*))W\right]g(\rho)d\rho\end{array}\right)$$
$$+\frac{1}{2}\left(\begin{array}{l}\left(W^C-W\right)\cdot\underbrace{\left(a(\underline{\Delta}^*)-a(\overline{\Delta}^*)\right)}_{<0}\cdot\underbrace{\left(G\left(\widehat{\rho}_{\overline{e}}(\underline{\Delta}^*)\right)-G\left(\widehat{\rho}_{\overline{e}}\left(\overline{\Delta}^*\right)\right)\right)}_{>0} \\ +\underbrace{\int_{\widehat{\rho}_{\overline{e}}(\underline{\Delta}^*)}^{\widehat{\rho}_{\overline{e}}(\overline{\Delta}^*)}\left[\rho\overline{W}+(1-\rho)W\right]g(\rho)d\rho}_{<0}\end{array}\right)$$

The only term that needs signing is $\int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^{1}\left[\rho\underline{W}+(1-\rho)W-a(\overline{\Delta}^*)W^C-(1-a(\overline{\Delta}^*))W\right]g(\rho)d\rho$.

First, since $\left[a(\Delta)W^C+(1-a(\Delta))W-\widehat{\rho}_{\underline{e}}(\Delta)\underline{W}-(1-\widehat{\rho}_{\underline{e}}(\Delta))W\right]<0$ for $k\geq\widetilde{k}$ (point (i) and Proposition 1), and also $\frac{\partial}{\partial\Delta}\left(MB(\overline{e})-MB(\underline{e})\right)>0$ (Lemma 3), then it is possible to have $\left[\rho\underline{W}+(1-\rho)W-a(\Delta)W^C-(1-a(\Delta))W\right]<0$ for $k<\widetilde{k}$.

Still, $\int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^{1}\left[\rho\underline{W}+(1-\rho)W-a(\overline{\Delta}^*)W^C-(1-a(\overline{\Delta}^*))W\right]g(\rho)d\rho$ may stay positive, because $\left[\rho\underline{W}+(1-\rho)W-a(\Delta)W^C-(1-a(\Delta))W\right]$ increases with ρ.

Nonetheless, since $\frac{\partial}{\partial\underline{W}}\left[\rho\underline{W}+(1-\rho)W-a(\overline{\Delta}^*)W^C-(1-a(\overline{\Delta}^*))W\right]>0$, for low enough $\underline{W}$ one has that $\int_{\widehat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^{1}\left[\rho\underline{W}+(1-\rho)W-a(\overline{\Delta}^*)W^C-(1-a(\overline{\Delta}^*))W\right]g(\rho)d\rho<0$.

Thus, a low enough $\underline{W}$ is a sufficient conditions for the existence of $k^*\in\left(0,\widetilde{k}\right)$ such that $W(\underline{e},\underline{\Delta}^*(\underline{e}))-W(\overline{e},\overline{\Delta}^*(\overline{e}))=0$.

The figure below gives the profile of the maximum welfare functions according to the above results. Proposition 2 merely states the obvious result of the comparison between $W(\underline{e},\underline{\Delta}^*(\underline{e}))$ and $W(\overline{e},\overline{\Delta}^*(\overline{e}))$ for the different levels of k.