Political Bargaining under Democracy and Autocracy.*

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March 6, 2008

Abstract

Models of elections tend to predict that parties will maximize votes by converging to an electoral center. There is no empirical support for this prediction. In order to account for the phenomenon of political divergence, this paper offers a stochastic electoral model where party leaders or candidates are differentiated by differing valences-the electoral perception of the quality of the party leader. If valence is simply intrinsic, then it can be shown that there is a "convergence coefficient", defined in terms of the empirical parameters, that must be bounded above by the dimension of the space, in order for the electoral mean to be a Nash equilibrium.

The idea of valence is then extended to include the possibility that activist groups contribute resources to their favored parties in reponse to policy concessions from the parties. The equilibrium result is that parties, in order to maximize vote share, must balance a centripetal electoral force against a centrifugal activist effect. Finally this model is applied to the case of a non-democratic regime, where an autocrat must bargain with economic and military elites in order to stay in power.

1 Introduction: Modeling Elections

The early work in modeling elections focused on two-party competition, and assumed a one-dimensional policy space, W, and "deterministic" voter choice. The models showed the existence of a "Condorcet" point, unbeaten under majority rule vote, at the median of the electoral distribution. Such models implied that

*This paper is based on work supported by NSF grant 0715929, and was completed while Schofield was Leitner visiting professor at Yale University. Prepared for presentation at ESNIE, Cargese, Corsica, May 2008.

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there would be strong centripetal political forces causing parties to converge to the electoral center (Hotelling, 1929; Downs, 1957; Riker and Ordeshook, 1973). In higher dimensions, such two party "pure strategy Nash equilibria" (PNE) generally do not exist, so the theory did not cover empirical situations where two or more policy dimensions were relevant.[1] It has been shown, however, that there would exist *mixed* strategy Nash equilibria (Kramer, 1978) whose support lies within a subset of the policy space known as the *uncovered set*.[2] "Attractors" of the political process, such as the uncovered set or a similar concept known as the Banks set (Penn, 2006) tend to be centrally located with respect to the distribution of voters' ideal points. The theoretical prediction that political candidates converge to the center is very much at odds with empirical evidence from U.S. presidential elections that political candidates do not locate themselves at the electoral center. [3]

The deterministic electoral model is ill-suited to deal with the multiparty case (where multiparty refers to the situation with three or more candidates). Recent work has focused on "stochastic" models which are, in principle, compatible with empirical models of voter choice [4] In such models, the behavior of each voter is modeled by a vector of choice probabilities.[5] Again, the usual result in this class of stochastic models is that all parties converge to the political center, in this case the electoral mean [6]

Empirical estimates of party positions in European multiparty polities can be constructed on the basis of various techniques of content analysis of party manifestos.[7] More recent analyses have been based on factor analysis of electoral survey data to obtain a multidimensional description of the main political issues in various countries. All these empirical analyses have obtained policy spaces that are two dimensional. These techniques allow for the estimatation of the positions of the parties in the empirically inferred policy space.[8] These estimates, together with related analysis using other estimation techniques have found no general tendency for parties to converge to the center.[9]

The lack of convergence is of particular concern in light of models of bargaining over coalition government in multiparty polities where no party has a majority. Consider the post-election phase of coalition bargaining, where the positions of the parties in the policy space W are assumed to be given by a vector, $\mathbf{z}$. The post-election distribution of seats defines a set of winning coali-

[1] See the surveys in Schofield 1985; Austen-Smith and Banks 1999.

[2] McKelvey (1986), Cox 1987; Banks, Duggan and Le Breton 2002, 2006.

[3] Poole and Rosenthal 1984; Schofield, Miller and Martin 2003.

[4] Schofield, Martin, Quinn and Whitford 1998; Quinn, Martin and Whitford 1999.

[5] Hinich 1977; Lin, Enelow and Dorussen 1999.

[6] Banks and Duggan 2005; McKelvey and Patty 2006.

[7] The original manifesto group of Budge *et al.* 1987 studied party policy in nineteen democracies. The more recent work of Budge, Klingemann *et al.* 2001 covers twenty five countries. See also Laver and Budge 1992, Laver and Hunt 1992, and Benoit and Laver 2006 who use expert estimates. Laver and Garry 2000 use content analysis of political texts.

[8] For Britain, Israel, Netherlands and the United States see Schofield and Sened 2006. For Italy see Giannetti and Sened 1994. For Argentina see Schofield and Cataife 2007; for Turkey see Schofield and Ozdemir 2008. .

[9] See Adams 1999a,b 2001: Adams and Merrill 1999.

tions, which we denote by $\mathbb{D}(\mathbf{z})$. Given the set of winning coalitions, and party positions, there are a number of bargaining models that have been proposed to account for government formation.[10] These models all suggest that if a policy position, known as the *core*, $Core(\mathbb{D}(\mathbf{z}), \mathbf{z})$, of the post -election bargaining game is non-empty, then this will be the outcome. McKelvey and Schofield (1987) obtained certain *pivotal symmetry conditions* that are necessary at a core point. If the "core" is stable under small perturbations in the positions of the parties then it is said to be "structurally stable." If a party's position is at the "structurally stable core", then we shall call this party the " core party". Laver and Schofield (1990) argued that under the circumstance of a structurally stable core, then the core party would be in a position to form a minority government, and take all perquisites of government for itself.

The next section of the paper considers the post election bargaining game in Israel for a sequence of elections from 1988 to 2006. The analysis suggests that after the election of 1992, the Labor party under Itzhak Rabin was located at the core position, and was able to form a minority government. Moreover, after the election of 2006, a center party called Kadima, formed originally by Ariel Sharon, was again at the core position. Although Kadima first formed a coalition government with two other parties, Labor and Israel Beiteinu, it was able to maintain itself even after Israel Beiteinu left the coalition.

In general if a core does exist, then it will be located close to the center of the electoral distribution. The implication that a party at a core position is able to dominate coalition bargaining in this way raises the question why all parties do not converge to an electoral center. For example, let z_0 denote the mean of the distribution of voter ideal points. [11] According to the standard stochastic model, all parties should converge to t the same position, z_0. With p parties, each one should receive the same proportion ,$\frac{1}{p}$, of the vote share. Each party should have the same probability of being in government, and each should receive the same share of government perquisites. Models of post-electionbargaining are then redundant. However, no empirical analyses have ever observed such a situation.

Section 3 of this paper will present a formal model of election which makes clear why the convergence result for the stochastic model need not hold. We assume that each party leader[12] adopts a position so as to maximize the share of the electoral vote. We also assume that there is an asymmetry in the electoral perception of the "quality" of party leaders (Stokes, 1992). The average weight given to the perceived quality of the leader of the j^{th} party is called the party's *valence.* In empirical models this valence is assumed to be exogenous, so it is independent of the party's position. Valence terms add to the statistical significance of the model. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future (Penn, 2003). Theorem 2 in Section 3 presents the necessary and sufficient conditions under which the joint

[10] Baron and Ferejohn 1989;Banks and Duggan 2000.

[11] This point can be estimated from a sample survey of the voter preferences.

[12] Although we speak of party leaders, the model is applicable to candidates for office as in U.S. presidential primaries or elections.

electoral mean, z_0, is a "local pure strategy Nash equilibrium" (LNE) of the stochastic vote model with exogenous valence.

The recent empirical work on stochastic vote models for a number of countries have been based on the "multinomial logit" assumption that the stochastic errors had a "Type I extreme value distribution" (Dow and Endersby, 2004), and have concluded that divergence rather than convergence is typical. With the same stochastic distribution assumption, Theorem 2 shows that a "convergence coefficient" incorporating all the parameters of the model, can be defined. This coefficient, c, involves the differences in the valences of the party leaders, and the "spatial coefficient" β. When the policy space,W, is assumed to be of dimension w, then the necessary condition for existence of an LNE at the electoral origin is that the coefficient c is bounded above by w. When the necessary condition fails, then parties, in equilibrium, will adopt divergent positions. In general, parties whose leaders have the lowest valence will take up positions furthest from the electoral mean. Moreover, because a pure strategy Nash equilibrium must be a local equilibrium, the failure of existence of LNE when all parties are at the electoral mean implies non existence of such a centrist PNE. The failure of the necessary condition for convergence has a simple interpretation. If the product of the spatial coefficient and the variance of the electoral distribution is sufficiently large in terms of the expected vote share of the lowest valence party at the electoral mean, then this party has an incentive to move away from the origin towards the electoral periphery. Other low valence parties will follow suit, and the local equilibrium will be one where parties are distributed along a "principal electoral axis." The general conclusion is that, with all other parameters fixed, then a convergent LNE can be guaranteed only when β is "sufficiently" small. Thus, divergence away from the electoral mean becomes more likely the greater is β, the valence difference and the variance of the electoral distribution.

To illustrate the theorem, Section 4 presents an empirical study of voter behaviour for Israel for the election of 1996. The estimated coefficients of the model are used to show that the condition on the empirical parameters of the model, necessary for convergence, was violated. The equilibrium positions obtained by simulation on the basis of the empirical stochastic model were found to be comparable with, though not identical to, the estimated positions: the two highest valence parties (Labor and Likud) were symmetrically located on either side of the electoral origin, while the lowest valence parties were located far from the origin.

Since vote maximization is a natural assumption to make for political competition under an electoral system based on proportional representation, the combination of empirical and formal analysis gives a plausible reason why convergence will not occur at some elections.

Section 4 continues with an empirical model for Turkey for the election of 2002. In 1999, the election was based on a proportional electoral system. More than eight parties contested the election, and the post election core was empty. The coalition government that formed was highly unstable and was unable to deal with the economic difficulties that faced the country. In 2002, in contrast, a high valence party, the Justice and Development Party, won 34% of the vote but

66% of the seats, indicating that the electoral method was highly majoritarian. This party was able to maintain its majority in the election of 2007.

To account for the disparity between the estimated positions of the parties in Israel and Turkey, and the local equilibrium positions obtained from the model with exogenous valence, the notion of valence is extended to incorporate the resources that party activists contribute to their chosen party. Theorem 1 in Section 3 then gives the first order condition for local equilibrium in this general model.The activist valence model is then applied in Section 5 to the non-democratic situation where.an autocrat must bargain with capitalist and military elite to obtain resources to maintain power. The formal result suggests that as the exogenous valence of the autocrat falls then he becomes more dependent on the resources supplied by his allies. In some cases, members of the capital elite may switch support from a low valence autocrat to provide resources for an anti-regime challenger.

The analysis of coalition behavior presented in Section 2 essentially takes the positions and strengths of the parties as given, and discusses the notion of a post election core, and its generalization, *the heart.* Section 3 then presents a formal stochastic model of voting based on exogenous valence and activist support for the parties. Section 4 presents the empirical analysis, while Section 5 extends the model to non-democratic regimes. Section 6 offers some conclusions based on the analysis.

2 Modeling Legislative Bargaining

We assume that each party chooses a preferred position (or *bliss point*) in a *policy space* W. From now on we shall denote the parties as $N=\{1,\ldots,j,\ldots,n\}$, and the vector of party positions as $\mathbf{z} = (z_1,\ldots,z_n)$. After the election we denote the number of seats controlled by party, j, by $s_j(\mathbf{z})$ and let $\mathbf{s}(\mathbf{z}) = (s_1(\mathbf{z}),\ldots,s_n(\mathbf{z}))$ be the of the vector of parliamentary seats. We shall suppose that any coalition with more than half the seats is winning, and denote the set of winning coalitions, at $\mathbf{z}$ by $\mathbb{D}(\mathbf{z})$. This assumption can be modified without any theoretical difficulty. For each winning coalition M in $\mathbb{D}(\mathbf{z})$ there is a set of points in W such that, for any point outside the set there is some point inside the set that is preferred to the former by all members of the coalition. Furthermore, no point in the set is unanimously preferred by all coalition members to any other point in the set. This set is the Pareto set of the coalition. If the conventional assumption is made that the preferences of the actors can be represented in terms of Euclidean distances, then this compromise set for a coalition is simply the convex hull of the preferred positions of the member parties. (In two dimensions, we can draw this as the area bounded by straight lines joining the bliss points of the parties and including all coalition members.) Since preferences are described by the vector, $\mathbf{z}$, we can denote this as $Pareto(M,\mathbf{z})$. Now consider the intersection of these Pareto sets for all winning coalitions, $\mathbb{D}(\mathbf{z})$. If this intersection is non-empty, then it is a set called the *Core* of $\mathbb{D}(\mathbf{z})$, written $Core(\mathbb{D}(\mathbf{z}),\mathbf{z})$. At a point in $Core(\mathbb{D}(\mathbf{z}),\mathbf{z})$ no coalition can propose an alternative policy point that

is unanimously preferred by every member of some winning coalition.

In general, $Core(\mathbb{D}(\mathbf{z}), \mathbf{z})$ will be at the preferred point of one party. A necessary and sufficient condition for point x to be in $Core(\mathbb{D}(\mathbf{z}), \mathbf{z})$ is that x is in the Pareto set of every minimal winning coalition [13] In the case that party preferences are derived from Euclidean utility functions in a two dimensional space, then the core will be non-empty if all median lines determined by the pair $(\mathbb{D}(\mathbf{z}), \mathbf{z})$ intersect. (Here a median line is simply a line through the positions of two parties such that a majority coalition is located on either side of the line.) When the core is empty then the heart, $\mathcal{H}(\mathbb{D}(\mathbf{z}), \mathbf{z})$, is defined to be the star shaped figure bounded by these median lines (or hyperplanes in higher dimension). These median lines can be identified with the preferred positions of a particular set of parties. These bounding "proto-coalitions" form the basis for coalitional bargaining. This model of the heart can then used to describe, heuristically, the general pattern of coalition formation.An attractive feature of the heart, regarded as a correspondence is that if $Core(\mathbb{D}(\mathbf{z}), \mathbf{z})$ is non empty, and the vector $\mathbf{z}'$ converges to $\mathbf{z}$ then there is a neighborhood of $\mathbf{z}$ within which $\mathcal{H}(\mathbb{D}(\mathbf{z}), \mathbf{z}')$ converges to $Core(\mathbb{D}(\mathbf{z}), \mathbf{z})$.[14]

The pre-election calculations of parties involve calculations over the relationship between party position, electoral response, and the effect that the resulting party positioning and parliamentary strength has on coalition bargaining. Schofield and Sened (2006) proposed that these calculations are based on beliefs by the political actors that can be represented by a "selection" from the heart correspondence. More formally, let $\mathcal{H}(\mathbb{D}(\mathbf{z}), \mathbf{z})$ represent the heart, when party positions are given by the vector, $\mathbf{z}$, and $\mathbb{D}(\mathbf{z})$ is the set of winning or decisive coalitions that occur after the election. Then beliefs can be represented by a mapping, $g : W^n \rightarrow \mathbb{W}$, where the selection $g(\mathbf{z})$ is a lottery with support, $\mathcal{H}(\mathbb{D}(\mathbf{z}), \mathbf{z})$ in W. This lottery, $g(\mathbf{z})$, in the space, $\mathbb{W}$, of all lotteries in W, specifies what party leaders expect to occur as a result of the choice of a vector $\mathbf{z} \in W^n$ of party positions, and the outcome, $\mathbb{D}(\mathbf{z})$. Schofield (2007a) suggested that the belief mapping, g, could be used as a formal device by which to model equilibrium choice of parties. However, the electoral mapping was not fully specified. The electoral model presented in Section 3 can, in principle be used to model g.

To illustrate the nature of the core and heart, consider the configuration of party strengths after the election of 1992 in Israel. (The election results in Israel for the period 1988 to 2003 are given in Table 1). The estimates of the voter positions in Figure 1 were obtained from factor analysis of a survey of the electorate carried out by Arian and Shamir (1995), complemented by an analysis of the party manifestos to obtain party positions. (The details can be found in Schofield and Sened, 2006.) The two dimensions of policy deal with attitudes to the PLO (the horizontal axis) and religion (the vertical axis).

[Insert Table 1 here]

As Figure 1 indicates, all median lines go through the Labor party position,

[13] A coalition in $\mathbb{D}(\mathbf{z})$ is minimal winning if it can lose no member and still belong to $\mathbb{D}(\mathbf{z})$.

[14] Technically the heart correspondence is lower hemi-continuous. See Schofield (1999).

so given the configuration of seats and positions, we can say Labor is the *core party* in 1992. Another way to see that the Labor position, z_{lab}, is at the core is to note that the set of parties above the median line through the Labor-Tsomet positions (but excluding Labor) only control 59 seats out of 120. The ability of the core party to control policy implies a tendency for core parties to form minority governments, since they need no other parties in order to fulfil their policy objectives. In fact, in 1992, Rabin first created a coalition government with Shas, and then formed a minority government without Shas. Given the vector ,$\mathbf{z}$, of party positions in Figure 1, and the electoral result, we can infer that $g(\mathbf{z}) = \{z_{lab}\}$.

We have emphasized that in two dimensions the core can be empty. To see the consequences of this, consider the configuration of party positions in Israel after the election of 1988, as presented in Figure 2, again using the seat allocations from Table 1, and factor analysis of voter attitudes (Arian and Shamir, 1990). In this case there is a median line through the Tzomet, Likud positions, so the coalition of parties above this line is winning. It is evident that the Labor does not belong to the Pareto set of the coalition including Likud, Tzomet and the religious parties. Indeed, it can be shown that the symmetry conditions (derived by McKelvey and Schofield 1987) necessary for the existence of a core are nowhere satisfied. In this case, there are cycles of different coalitions, each preferred by a majority of the legislature to some other coalition policy in the cycle.

[Insert Figures 1 and 2 here]

The heart, $\mathcal{H}(\mathbb{D}(\mathbf{z}), \mathbf{z})$, given the seat strengths and party positions, is the star-shaped figure, bounded by the five median lines. It is reasonable to conclude, in the absence of a core party, that coalition government will be based on a small number of minimal winning coalitions. The work of Banks and Duggan (2000) considers bargaining between political parties when the party positions and seat strengths are given. Following their analysis, Schofield (2007a) suggested that the outcome can be described as a *lottery,* $g(\mathbf{z})$, whose support is the star shaped heart. Note that the bounding coalitions of the heart include {Shas, Likud}, {Shas, Labor} and {Labor, Likud}. It is evident that Shas pivots between the two larger party. Even though Shas contolled few seats in 1988, it had significant bargaining power.

Figure 3 shows the positions of the parties after the election of 1996, together with an estimate of the electoral distribution, based on the survey data obtained by Arian and Shamir (1999). It is easy to show that the core, $Core(\mathbb{D}(\mathbf{z}), \mathbf{z})$ in 1996 is empty, as indeed it was after the election of 2003. In both years Shas was pivotal between coalitions led by Labor (in 1996) and Likud (in 2003).

[Insert Figure 3 here]

Figure 4 can be used to understand the consequences after Sharon seemingly changed his policy on the security issue in August, 2005, by pulling out of the Gaza Strip. First, members of Likud reacted strongly against this change in policy. Then, in the first week of November, 2005, Amir Peretz, a union activist, and leader of Am Ehad, won an election against Shimon Peres for leadership of the Labor Party.

Sharon then left the Likud Party and allied with Peres and other senior Labor Party members, to form the new party, *Kadima* ("Forward"). We can infer that the coalition of Sharon and Peres positioned Kadima at the origin of the policy space, as shown in Figure 4. This figure gives estimates of party positions at the March 28, 2006, election to the Knesset. Because of Sharon's stroke in January, 2006, Ehud Olmert had taken over as leader of Kadima, and was able to take 29 seats. Likud only took 12 seats, while the four parties on the upper right of the figure won 38 seats. One surprise of the election was the appearance of a Pensioners' party with 7 seats. As Figure 4 indicates, the parties on the right (even with the Pensioners' Party) do not have the required 61 seats for a majority, so Kadima is located at core position. It appears that Sharon's change of policy has led to a fundamental transformation in the political configuration, from the coalition structure without a core (that had persisted since 1996), to a new configuration, associated with the center, core party, Kadima. Even though Kadima is estimated to be a core party, Olmert needed the support of Labor to be able to deal with the complex issue of fixing a permanent border for Israel. The debacle in Lebanon severely weakened Olmert's popularity, and on October, the 61 members of the Kadima-Labor coalition voted to bring Israel Beiteinu into the coalition. The report, in April 2007, on the failure of the government during the war with Lebanon in Summer 2006 seemed to threatened the Kadima-Labor- Israel Beiteinu coalition by bringing about a change in the Labor party leadership. Ehud Barak won the Labor party leadership on June 13, while Shimon Peres became President. Although Labor stayed in the coalition, on January 15, 2008, Avigdor Lieberman, chairman of Israeli Beiteinu announced that the party would quit the government because of disagreement over issues such as Jerusalem, the refugees and the contours of a future Palestinian state. On February 3, 2008 Barak agreed to remain in the coalition, thus helping to sustain Kadima in power. Note that though Kadima appears to be at a core position, the core is not structurally stable. A small movement by Labor on the religion axis could create the possibility of a (somewhat unlikely) coalition of Labor with Likud and Israel Beiteinu

[Insert Figure 4 here]

Israel presents extremely complex configurations of party positioning and coalition bargaining. As a first effort at modeling the positioning of parties we now introduce the formal stochastic model.

3 A Model of Leader Support

The model presented here is an extension of the standard multiparty stochastic model, modified by inducing asymmetries in terms of valence.

The key idea underlying the formal model is that political leaders attempt to estimate the effects of their policy positions on the support they receive. Each leader, whether autocrat or opposition, chooses the policy position as best response to opposing position(s), in order to obtain sufficient support either to retain power or to gain power. The stochastic model essentially assumes that a

leader cannot predict support precisely, but can estimate an expected support. In the model with valence, the stochastic aspect of the model is associated with the weight given by each citizen, i, to the average perceived quality or valence of the party leader.

Definition 1. **The Stochastic Model $E(\boldsymbol{\lambda}, \boldsymbol{\mu}, \beta; \Psi)$ with Activist Valence**.

The data of the spatial model is a distribution, $\{x_i \in W : i \in P\}$, of voter ideal points for the members of the *selectorate*, P, of size p. By the selectorate we mean those citizens who have some potential to influence political choice. We assume that W is a open, convex subset of Euclidean space, $\mathbb{R}^w$, with w finite. Each of the leaders in the set $N = \{1, \ldots, j, \ldots, n\}$ chooses a policy, $z_j \in W$, to declare. Let $\mathbf{z} = (z_1, \ldots, z_n) \in W^n$ be a typical vector of leader positions.

Given $\mathbf{z}$, each citizen, i, is described by a vector

$$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_n))$$

where

$$u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta||x_i - z_j||^2 + \epsilon_j = u^*_{ij}(x_i, z_j) + \epsilon_j. \tag{1}$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. The term, λ_j, is the fixed or *exogenous valence* of leader j, while the function $\mu_j(z_j)$ is the component of valence generated by activist contributions to leader j. The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of the Euclidean metric, $||a - b||$, on W. The vector $\epsilon = (\epsilon_1, \ldots, \epsilon_j, \ldots, \epsilon_n)$ is the stochastic error, whose multivariate cumulative distribution will be denoted by Ψ.

It is assumed that the exogenous valence vector

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n) \text{ satisfies } \lambda_n \geq \lambda_{n-1} \geq \cdots \geq \lambda_2 \geq \lambda_1.$$

Citizen behavior is modelled by a probability vector. The probability that a citizen i chooses leader j at the vector $\mathbf{z}$ is

$$\begin{aligned} \rho_{ij}(\mathbf{z}) &= \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j]. & (2) \\ &= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \text{ for all } l \neq j] & (3) \end{aligned}$$

Here Pr stands for the probability operator generated by the distribution assumption on ϵ.

The *expected support* of leader j is.

$$V_j(\mathbf{z}) = \frac{\sum_{i \in P} s_{ij} \rho_{ij}(\mathbf{z})}{\sum_{i \in P} s_{ij}} \tag{4}$$

Because of the nature of electoral systems, the weight s_{ij} may differ between different voters i in P. In non-democratic polities the weights s_{ij} may differ widely.

The differentiable function $V : W^n \to \mathbb{R}^n$ is called the *leader profile function.*

For simplicity we shall assume that for each the j, the sum $\sum_{i \in P} s_{ij} = 1$. In democratic polities, for example, s_{ij} is usually assumed to be $\frac{1}{p}$ for all i, j. We call this the *egalitarian* case. In non-democratic polities the weights s_{ij} may differ widely.

In the following it is assumed that the stochastic errors have the Type I extreme value (or Gumbel) distribution,Ψ. The formal model based on Ψ parallels the empirical models based on multinomial logit (MNL) estimation.

Definition 2. The Extreme Value Distribution,Ψ.

The cumulative distribution,Ψ, has the closed form

$$\Psi(x) = \exp\left[-\exp\left[-x\right]\right].$$

The difference between the Gumbel and normal (or Gaussian) distributions is that the latter is perfectly symmetric about zero.

With this distribution assumption, it follows, for each voter i, and leader j, that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{n} \exp u^*_{ik}(x_i, z_k)}. \tag{5}$$

In this stochastic electoral model it is assumed that each leader j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_n)$.

Definition 3. Equilibrium Concepts.

(i) A strategy vector $\mathbf{z}^*=(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*) \in W^n$ is a *local strict Nash equilibrium* (LSNE) for the profile function $V : W^n \to \mathbb{R}^n$ iff, for each leader $j \in N$,there exists a neighborhood W_j of z_j^* in W such that

$$\begin{aligned} V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) &> V_j(z_1^* \ldots z_{j-1}^*, z_j, z_{j+1}^* \ldots, z_n^*) \\ \text{for all } z_j &\in W_j - \{z_j^*\}. \end{aligned}$$

(ii) A strategy vector $\mathbf{z}^*=(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*)$ is a *local weak Nash equilibrium* (LNE) iff, for each agent j,there exists a neighborhood W_j of z_j^* in W such that

$$\begin{aligned} V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*) &\geq V_j(z_1^* \ldots z_{j-1}^*, z_j, z_{j+1}^*, \ldots, z_n^*) \\ \text{for all } z_j &\in W_j. \end{aligned}$$

(iii) A strategy vector $\mathbf{z}^*=(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*)$ is a *strict* or *weak*, *pure strategy Nash equilibrium* (PSNE or PNE) iff W_j can be replaced by W in (i),(ii) respectively.

(iv) The strategy z_j^* is termed a "local strict best response," a "local weak best response," a "global weak best response," a "global strict best response," respectively to $\mathbf{z}^*_{-j}=(z_1^*, \ldots, z_{j-1}^*, z_{j+1}^*, \ldots, z_n^*)$. □

Obviously if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. We use the notion of LSNE to avoid problems

with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}^*$ to be a LNE and thus a PNE, without having to invoke concavity. Of particular interest is the vector

$$x_j^* = \frac{\sum_{i\in P} s_{ij}x_i}{\sum_{i\in P} s_{ij}} = \sum_{i\in P} s_{ij}x_i. \tag{6}$$

In section 5 below we consider the case where there is an autocrat, $j = auto$, in power,and we follow Bueno da Mesquita et al (2003) and call x^*_{auto} the *selectorate* origin. By a change of coordinates choose $x^*_{auto} = 0$.

In section 4 we consider the *egalitarian* case,where all $s_{ij} = 1/p$. Again we can again transform coordinates so that in the new coordinate system, $x^* = \frac{1}{p}\sum_{i\in P} x_i = 0$. For this case we shall refer to $\mathbf{z}_0 = (0,\dots,0)$ as the *joint electorate origin.*

Theorem 1 shows, even in the egalitarian case, that $\mathbf{z}_0 = (0,\dots,0)$ will generally not satisfy the first order condition for a LSNE, namely that the differential of V_j, with respect to z_j be zero. However, if the activist valence function is identically zero, so that only exogenous valence is relevant, then the first order condition at $\mathbf{z}_0$ will be satisfied.

It follows from the definition of the Gumbel distribution, that for voter i, with ideal point, x_i, the probability, $\rho_{ij}(\mathbf{z})$, that i picks j at $\mathbf{z}$ is given by

$$\rho_{ij}(\mathbf{z}) = [1 + \Sigma_{k\neq j}[\exp(f_{jk})]]^{-1} \tag{7}$$

$$\text{where } f_{jk} = \lambda_k + \mu_k(z_k) - \lambda_j - \mu_j(z_j) + \beta||x_i - z_j||^2 - \beta||x_i - z_k||^2.$$

Schofield (2006a) shows that the first order condition for $\mathbf{z}^*$ to be a LSNE is that it be a *balance solution.*

Definition 4. The balance solution for the model $E(\boldsymbol{\lambda}, \boldsymbol{\mu}, \beta; \Psi)$.

Let $[\rho_{ij}(\mathbf{z})] = [\rho_{ij}]$ be the matrix of voter probabilities at the vector $\mathbf{z}$**,** and let

$$\alpha_{ij} = \frac{s_{ij}[\rho_{ij} - \rho_{ij}^2]}{\sum_{k\in P} s_{kj}[\rho_{kj} - \rho_{kj}^2]}. \tag{8}$$

be the matrix of coefficients. The *balance equation* for z_j^* is given by expression

$$z_j^* = \frac{1}{2\beta}\frac{d\mu_j}{dz_j} + \sum_{i=1}^{p} \alpha_{ij}x_i. \tag{9}$$

The vector $\sum_i \alpha_{ij}x_i$ is called the *weighted electoral mean* for leader j,and can be written

$$\sum_{i=1}^{p} \alpha_{ij}x_i = \frac{d\mathcal{E}_j^*}{dz_j}. \tag{10}$$

Notice first that the weight α_{ij} shows how the citizen i influence leader j in his choice of policy position. Moreover, the weights for leader j depend on the

vector of positions $\{\mathbf{z}_{-j}\}$ of leaders other than j. The balance equation can be rewritten as

$$\left[\frac{d\mathcal{E}_j^*}{dz_j} - z_j^*\right] + \frac{1}{2\beta}\frac{d\mu_j}{dz_j} = 0. \tag{11}$$

The bracketed term on the left of this expression is termed the *marginal electoral pull of leader* j and is a gradient vector pointing towards this leader's weighted electoral mean. This position is that point where the electoral pull is zero. The vector $\frac{d\mu_j}{dz_j}$ is called *the marginal activist pull for leader* j.

If $\mathbf{z}^* = (z_1^*, ..z_j^*, ..z_n^*)$ is such that each z_j^* satisfies the balance equation, then call $\mathbf{z}^*$ the *balance solution.*

Theorem 1. (Schofield, 2006a)

Consider the electoral model $E(\boldsymbol{\lambda}, \boldsymbol{\mu}, \beta; \Psi)$ based on the Type I extreme value distribution, and including both exogenous and activist valences. The first order condition for $\mathbf{z}^*$ to be an LSNE is that it is a balance solution. If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then the balance solution will be a PNE.

We emphasize that the *marginal electoral pull of leader* j is a gradient vector pointing towards the weighted electoral mean of the leader, and represents the *centripetal* pull to the center. *The marginal activist pull for leader* j represents the *centrifugal force* generated by the resources made available by activists.

In principle, this model can be used to examine the equilibrium position of a political leader, responding to activist demands, and balancing the pull of the selectorate, in order to gain resources that can be used to compete with political opponents. Even without activists, convergence to a centrist position, as in the Downsian model, is impossible if the population is sufficiently heterogenous in its beliefs or preferences.

In the case $\mu_j = 0$ for all j, then the balance condition becomes

$$z_j = \sum_{i \in P} s_{ij} x_i. \tag{12}$$

In the egalitarian case with all weights $\{s_{ij}\}$ identical, then first order balance condition becomes

$$z_j^* = \frac{1}{p}\sum_{i=1}^{p} x_i. \tag{13}$$

By a change of coordinates we can choose $\frac{1}{p}\Sigma x_i = 0$. In this case, the marginal electoral pull is zero at the origin and the joint origin $\mathbf{z}_0 = (0, \ldots, 0)$ satisfies the first order condition. However, since $\boldsymbol{\mu} = \mathbf{0}$, we cannot use the concavity of $\boldsymbol{\mu}$ to assert the existence of equilibrium. Schofield (2007) shows that if $\boldsymbol{\mu} = \mathbf{0}$, then there is a coefficient, c, defined in terms of all model parameters and the electoral covariance matrix of the voter preferred points such that $c < w$ is a necessary condition for $\mathbf{z}_0$ to be a LSNE in the egalitarian stochastic vote model.

Definition 5. The Electoral Covariance Matrix, ∇_0.

Let $W = \mathbb{R}^w$ be endowed with a system of coordinate axes $r = 1, \ldots, w$. For each coordinate axis let $\xi_r = (x_{1r}, x_{2r}, \ldots, x_{pr})$ be the vector of the r^{th}

coordinates of the set of p voter bliss points. The scalar product of ξ_r and ξ_s is denoted $(\xi_r \centerdot \xi_s)$.

(i) The symmetric $w \times w$ *electoral covariance matrix* about the origin is denoted ∇_0 and is defined by

$$\nabla_0 = \frac{1}{p}\left[(\xi_r \centerdot \xi_s)\right]_{s=1,\ldots,w}^{r=1,\ldots,w}.$$

(ii) Let $(\sigma_r, \sigma_s) = \frac{1}{p}(\xi_r, \xi_s)$ be the electoral covariance between the r^{th} and s^{th} axes, and $\sigma_s^2 = \frac{1}{p}(\xi_s, \xi_s)$ be the electoral variance on the s^{th} axis, with

$$\sigma^2 = \sum_{s=1}^{w} \sigma_s^2 = \frac{1}{p}\sum_{s=1}^{w}(\xi_s \centerdot \xi_s) = trace(\nabla_0)$$

the total electoral variance.

Theorem 2. (Schofield, 2007b)

(i)The Hessian of the egalitarian vote share function of party j at $\mathbf{z}_0$ is a positive multiple of the w by w matrix.

$$C_j = 2\beta(1 - 2\rho_j)\nabla_0 - I \tag{14}$$

where I is the w by w identity matrix.

(ii) The necessary and sufficient condition for $\mathbf{z}_0$ to be an LSNE is that all C_j have negative eigenvalues. Since C_1 must also have negative eigenvalues, it follows that a necessary condition for $\mathbf{z}_0$ to be an LNE is that a convergence coefficient, c, defined by

$$c = 2\beta(1 - 2\rho_1)\sigma^2$$

is bounded above by the dimension, w.

(iii) In two dimensions, a sufficient condition is that c is bounded above by 1. In higher dimensions a sufficient condition can be expressed by appropriate bounds on the cofactors of C_1.

While maximization of vote share is an appropriate maximand under proportional egalitarian rule, a more appropriate maximand under plurality rule would be a seat share function

$$S_j(\mathbf{z}) = S_j(V_1(\mathbf{z}), ..., V_j(\mathbf{z}), ...V_n(\mathbf{z}))$$

which might very well be a logistic function of $V_j(\mathbf{z})$. The results given in Theorems 1 and 2 can be extended to this case.

4 Empirical Analyses

4.1 Elections in Israel

Schofield and Sened (2006) estimated a multinomial logit model using the assumption of a Type I extreme value distribution on the errors.[15] Using the formal analysis presented above, we can readily show that the *convergence coefficient* of the model greatly exceeds 2 (the dimension of the policy space). Indeed, one of the eigenvalues of the Hessian of the low valence party, the NRP (also called Mafdal), can be shown to be positive. Indeed it is obvious that there is a principal component of the electoral distribution, and this axis is the eigenspace of the positive eigenvalue. It follows that low valence parties should position themselves close to this principal axis, as illustrated in the simulation given below in Figure 5

The MNL estimation of the stochastic model shows that in 1996, the lowest valence party was the NRP with valence -4.52, while Labor had the highest valence of 4.15, and the Likud valence was 3.14. The spatial coefficient was $\beta = 1.12$, so to use the previous reult, we note that the valence difference between the NRP and Labor was $4.15 - (-4.52) = 8.67$, while the difference between the NRP and Likud was $3.14 - (-4.52) = 7.66$. Since the electoral variance on the first axis is 1.0, and on the second axis it is 0.732, with covariance 0.591, we can compute the characteristic matrix of the NRP at the origin as follows:

$$\begin{aligned} \rho_{NRP} &\simeq \frac{1}{1 + e^{4.15+4.52} + e^{3.14+4.52}} \simeq 0. \\ A_{NRP} &\simeq \beta = 1.12. \\ C_{NRP} &= 2(1.12) \begin{pmatrix} 1.0 & 0.591 \\ 0.591 & 0.732 \end{pmatrix} - I = \begin{pmatrix} 1.24 & 1.32 \\ 1.32 & 0.64 \end{pmatrix}. \end{aligned}$$

From the estimate of C_{NRP} it follows that the two eigenvalues are 2.28 and -0.40, giving a saddlepoint, and a value of 3.88 for the convergence coefficient. This exceeds the necessary upper bound of 2. The major eigenvector for the NRP is (1.0, 0.8), and along this axis the NRP vote-share function increases as the party moves away from the origin. The minor, perpendicular axis associated with the negative eigenvalue is given by the vector $(1, -1.25)$. The simulation of the model to obtain local equilibria made it clear that the local equilibrium positions of all parties lay on a principal axis through the origin and the point (1.0, 0.8). In all, five different LNE were located. However, in all equilibria, the two high valence parties, Labor and Likud, in Figure 4 were located close to the simulated equilibrium positions. The only difference between the various equilibria was that the positions of the low valence parties were perturbations of each other.

[15] This estimated model correctly predicts 63.8 percent of the voter choices. The log marginal likelihood of the model was -465.

It is evident that if the high valence party occupies the electoral origin, then all parties with lower valence can compute that their vote-share will increase by moving up or down the principal electoral axis. In seeking local maxima of the vote shares all parties other than the highest valence party should vacate the electoral center. Then, however, the first-order condition for the high valence party to occupy the electoral center would not be satisfied. Even though this party's vote-share will be little affected by the other parties, it too should move from the center. The simulation for 1996 is compatible with the formal analysis: low valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral center. Their optimal positions will lie either in the "north-east" quadrant or the "south-west" quadrant. The vote-maximizing model, without any additional information, cannot determine which way the low valence parties should move. The simulation of the local Nash equilibria ilalso made clear the inverse relationship between a party's valence and the distance of the party's equilibrium position from the electoral mean. A similar analysis is given in Schofield and Sened (2006) for the elections of 1992 and 1988. In 1988 the two eigenvalues for Shas were +2.0 and -0.83,while in 1992 the eigenvalues for this party were +2.12 amd -0.52. Just as in 1996, the theoretical model of vote maximization implies that all parties should be located on a principal electoral axis

In contrast, since the valence difference between Labor and Likud was relatively low, their local equilibrium positions are close to, but not identical to, the electoral mean. Intuitively it is clear that once the low valence parties vacate the origin, then high valence parties, like Likud and Labor, should position themselves almost symmetrically about the origin, and along the principal axis. Although we have not performed the empirical analysis for the elections of 2003 and 2006, we can expect the same result to hold.

There is a disparity between the estimated party positions in 1996 given in Figure 3 and the simulated equilibrium positions. The two religious parties, Shas and Yahadut, are estimated to be far from the principal axis, seeming in contradiction to the prediction of the stochastic model. Moreover, the high valence parties, Labor and Likud appear further from the origin than suggested by the simulation. In 2006, it is plausible that Kadima was able to position itsels at the electoral center was precisely because of the extremely high valence of Ariel Sharon.

The disparity between the prediction of the nature of local equilibria in the stochastic model with exogenous valence and the actual postions of the parties may be accounted for by using the more general model that allows valence to be influenced by party activists.

4.2 Elections in Turkey 1999-2007

In empirical analysis it is difficult to estimate the activist valence functions. However, it is possible to use socio-demographic variables as proxies. Instead of

using (1) as the estimator for voter utility, we can use the expression

$$u_{ij}(x_i, z_j) = \lambda_j - \beta\|x_i - z_j\|^2 + \theta_j^{\mathrm{T}}\eta_i + \varepsilon_j. \tag{15}$$

where the k -vector θ_j represents the effect of the k different socio-demographic parameters (class, domicile, education, income, etc.) on voting for party j while η_i is a k-vector denoting the i^{th} individual's relevant "socio-demographic" characteristics. We use θ_j^{T} to denote the transpose of θ_j so $\theta_j^{\mathrm{T}}\eta_i$ is a scalar. When β and $\{\lambda_j\}$ are assumed zero then we call the model pure *socio-demographic* (SD). When $\theta_j^{\mathrm{T}}\eta_i$ are assumed zero then the model is called *pure spatial*, and when all parameters are included then the model is called *joint*. The differences in log marginal likehoods for two models then gives the Log Bayes' factor for the pairwise comparison.[16] This technique can then be used to determine which is the superior model. We can use this model to explain Turkish election results in 1999 and 2002, given in Tables 2 and 3. Figures 6 and 8 show the electoral distributions (based on sample surveys of sizes 635 and 483, respectively) and estimates of party positions for 1999 and 2002.

Minor differences between these two figures include the disappearance of the Virtue Party (FP) which was banned by the Constitutional Court in 2001, and the change of the name of the Kurdish party from HADEP to DEHAP. The most important change is the appearance of the new Justice and Development Party (AKP) in 2002. This latter party obtained about 35 percent of the vote and 363 seats out of 550 seats in 2002. Figure 7 presents an estimate of the heart in 1999. In 1999, a DSP minority government formed, supported by ANAP and DYP. This only lasted about 4 months, and was replaced by a DSP-ANAP-MHP coalition. During the period 1999–2002, Turkey experienced two severe economic crises. As the tables show, the vote shares of the parties in the governing coalition went from about 64 percent in 1999 to 15 percent in 2002. In 2002, AKP obtained a majority of the seats, as a result of the of the use of a majoritarian electoral system. Schofield and Ozdemir (2008) estimated a MNL model of these elections.[17] The estimations include various socio-demographic characteristics such as religious orientation. Note in particular that one of the socio-demographics is "Alevi." Alevis belong to a non-Sunni religious/cultural community living in Turkey and in some parts of Iran, and comprise 13-15 percent of the Turkish population. They are closer to Shia Islam than Sunni Islam but the majority of Shia and Sunni do not regard Alevis as Muslims. Alevis tend to support "Kemalism" and the secular state and vote for CHP, leading to a Sunni-Alevi tension in the society. [Insert Tables 2 and 3 and Figures 6, 7 and 8 here]

The Log Bayes' factors showed that the joint MNL model was superior to all others. It is noticeable that the valences of the ANAP and MHP dropped between 1999 and 2002. In 1999, the estimated λ_{ANAP} was -0.114, whereas

[16] Since the Bayes' factor for a comparison of two models is simply the ratio of marginal likelihoods, the log of the Bayes factor is the difference in log likelihoods.

[17] The estimation is based on a factor analysis of a sample survey conducted by Veri Arastima for TUSES.

in 2002 it was -0.567, while λ_{MHP} fell from 2.447 to 1.714. The estimated valence, λ_{AKP}, of the new Justice and Development Party (AKP) in 2002 was 1.968, which we might ascribe to the disillusion of most voters with the other parties, as well as the charisma of Recep Tayyip Erdogan, leader of the AKP.[18] The β coefficient was 0.456 in 1999, and 1.445 in 2002, suggesting that electoral preferences over policy had become more intense. The estimated convergence coefficient, c, is 2.014 for 1999 and 6.48 in 2002, giving a formal reason why convergence should not occur in these elections

To compute the convergence coefficient for 2002, we proceed as follows.

4.2.1 The 2002 Election

In 2002, the electoral variance on the first axis (religion) was estimated to be 1.18 while the electoral variance on the second axis was 1.15, with the covariance between the two axes equal to 0.74.

Thus

$$\nabla_0 = \begin{bmatrix} 1.18 & 0.74 \\ 0.74 & 1.15 \end{bmatrix}$$

with $trace(\nabla_0) = 2.33$.
The β coefficient was 1.445, while the party with the lowest valence is ANAP with $\lambda_{ANAP} = -0.567$.
The probability, ρ_{ANAP}, that a voter chooses ANAP is given by

$$\begin{aligned} & [1 + \exp(2.535) + \exp(1.67) + \exp(3.163) + \exp(2.281)]^{-1} \\ = \quad & 0.019, \end{aligned}$$

The Hessian of the vote share function of ANAP (when all parties are at the origin is

$$\begin{aligned} C_{CHP} &= \begin{bmatrix} 2.28 & 2.06 \\ 2.06 & 2.20 \end{bmatrix}. \\ \text{and } c &= 2 \times 1.39 \times 2.33 = 6.48. \end{aligned}$$

This greatly exceeds the upper bound of 2.0 for convergence to the electoral origin. The major eigenvalue for the ANAP is 4.30, with eigenvector $(+1.10, +1.0)$ while the minor eigenvalue is 0.18, with orthogonal eigenvector $(-1.0, +1.10)$. In this case, the electoral origin is a minimum of the vote share function of ANAP. As before, the first eigenvector corresponds to the principal electoral component, or eigenspace, alligned at approximately 45 degrees to the religion axis. On both principal and minor axis, the vote share of ANAP increases as it moves away from the electoral origin, but because the major eigenvalue is much larger than the minor, we can expect some of the parties in equilibrium to adopt positions far from the principal electoral axis. Figure 8 is consistent with this inference. [Insert Table 4 about here]

[18] Abdullah Gul became Prime Minister after the November 2002 election because Erdogan was banned from holding office. Erdogan took over as Prime Minister after winning a by-election in March 2003.

In the 2007 election, the Kurdish Party (now called the Freedom and Solidarity Party, DTP) contested the election as independents, and thus were not subject to the 10 percent cut-off, and were able to win 24 seats. As Table 4 shows, the AKP took 46.6 percent of the vote and 340 seats (or 61.8%), reflecting the continuing high valence of Erdogan. Abdullah Gul, Erdogan's ally in the AKP and a practising Muslim who has been Turkey's foreign minister for over four years, was elected as the country's 11th president on 28 August, despite strong opposition from the army and militant secularists.[19] Tension between the military, the AKP and the Kurdish population has increased since the election.

Notice that the election results of 1999 were based on an electoral system that was quite proportional, whereas in 2002 and 2007, the electoral system was highly majoritarian. In 2002, the AKP gained 66% of the seats with only 35% of the vote. This suggests that plurality electoral systems magnify the effect of party activists.

5 Extension of the model

Recent work by Acemoglu and Robinson (2006) and Przeworski et al. (2000) have explored the transition from autocratic regimes to democracy. Indeed in Latin America there have been waves of democraizarion and then reversion to military or autocratic rule. In this section we briefly comment on the application of the activist model to such political transitions.

In the previous examples drawn from Israel and Turkey, the nature of the policy space was drawn fron voter surveys, and based on religion and nationalism. To construct a general theoretical model, we first start with the political economic assumption that power derives from the control of the factors of capital, land and labor. The distribution of these factors can be desribed by a point in a high dimensional *economic factor space*. For purposes of exposition, Figure 9 gives an extreme simplification of this idea, representing the factor space as a horizontal axis with Labor/Land at one end and Capital at the other. Perpendicular to the economic space is the *political space.* Again, for puposes of exposition, we can assume this space is uni-dimensional. In modern democracies, this axis can be identified with civil and social rights. [Insert Figure 9 here]

The idea underlying this figure follows from the work of North and Weingast (1989), as applied by Schofield (2006b).

This work suggests that the political economic equilibrium in a society is the result of a bargain between the elite holders of factors, and those who govern the institutions. A political leader, whether democratically elected, or holding onto power by force, must have enough support from the elite or the people, or both, to stay in power. For example, Schofield 2006b) argues that the key compact between land, capital and the Whigs in Britain was initiated in 1720, and involved the protection of land via increased customs and excise. This

[19] *The Independent,* 29 September 2007.

enabled the government of Britain to dramatically increase its borrowing so as to prosecute war with France, but required maintaing the restriction of the franchise. The formal model of power presented in Schofield (2008) has the following features:

Firstly, each factor elite has an ellipsoidal utility function, as illustrated in Figure 9, indicating their primary concern with that factor. Similarly the political elite, whether autocrat or prime minister or president, is less interested in the particular disposition of factors, but rather in their utilization in order to maintain power. This assumption on elite utilities allows the economic and political elite to bargain. Figure 8 presents a *contract curve* between the economic elite (whether land or capital) and the autocrat's supporters, representing the set of bargains that are possible. In many parts of the world, the key autocrat supporters would be the military. It is implicit here that the preferred societal policy point of different elements of the capital elite do not coincide with those of the military. This contract curve specifies the nature of the resources, military and capitalistic, that can be made available to the political leader. Again, it is not crucial that the bargain be only between capital and the political or military elite. It is quite possible in some regimes that the landed elite control the critical factor. The resources made available by this contract can then used to maintain political power. The activist valence of the autocrat can then be expressed as a combination

$$\mu_{auto}(z_A) = \mu_A(\Sigma_A(u_A(z_A))) + \mu_C(\Sigma_C(u_C(z_C))). \tag{16}$$

Here $\Sigma_A(u_A(z_A))$ are the resources contributed by the immediate autocrat supporters, expressed in terms of the utility function, $u_A(z_A)$, dependent on the autocrat position, while $\Sigma_C(u_C(z_C))$ are the resouces contributed by the capitalist elite. In the same way we may assume that anti-regime leaders will gain resources from democratic and labor activists, as described by an opposed contract curve between the points L and S in Figure 8. In the formal model, each member of the population has a utility function, based partly on some preferred position in the factor space, but also on what we have called the *valences* of the various political leaders. This model distinguished between exogenous (or intrinsic) valence and the valence that results from the resouces made available to the political leader by the factor elite. While the contract curve specifies the locus of actions that maximizes resources, the *balance locus* gives the equilibrium locus of the political leader. In a democratic regime, this will depend on the intrinsic valences of political opponents and the activist contributions. In the model of autocracy, the equilibrium position of the leader will be a weighted sum of the preferred positions of those with some power in the polity (the selectorate). In both models, the leader with greater intrinsic valence will be less dependent on the resource support of activists or the factor elite. Moreover, the greater the intrinsic valence of an opponent, whether a revolutionary or a leader of a democratically chosen opposition, the further will the leader's position be from the center. In Figure 10, the point denoted "the mean of the selectorate" is used to denote the center. The expression given in (16) is for the simple case of

two activist groups supporting the autocrat. In fact the model can be readily generalized to the case of many groups. In this, one can use the concept of the heart to delinate the bargaining domain of the various political groups.

One inference from this model is that the "equilibrium" position of the autocrat may be so far from the center that the populace will be induced to revolution. On the other hand, some authoritarian systems have evolved so that the "autocratic equilibrium" is stable. While we cannot overlook culture and historical distinctiveness, the authoritarian government's institutional design almost certainly contributes to its relative durability. By applying the model just proposed, it may be possible to pinpoint which authoritarian systems are more durable and why. Schofield and Levinson (2008) used this model to examine three types of authoritarian regimes that have predominated in the twentieth-century: bureaucratic military dictatorship, fascist dictatorship, and the communist party dictatorship.

Their work suggests that the military bureaucratic regime are the least durable, while fascist dictatorship was more durable. The socialist party dictatorship was very long lived, but eventually crashed in dramatic fashion. Recent events in Russia suggest that a form of autocracy has reappeared.

Schofield and Levinson showed how the theoretical prerequisites for regime change to democracy were sequentially harder to meet. The prerequisites can include:

(1) enough economic and or political inequality to induce an oppositional underclass to demand formally institutionalizing some power redistribution.

(2) not so much inequality in economic or political power that the authoritarian elite is willing to incur almost any cost to keep power.

(3) the ability of the regime's opponents to overcome the collective action problem inherent in organizing a revolution.

(4) for democracy to be achieved, reformers within the authoritarian bloc must align themselves with moderate opposition leaders to force authoritarian hardliners into accepting transition.

6 Concluding remarks

This paper has applied a theoretical stochastic model of elections in Israel and Turkey, in order to gain some insight into the complexities of multi-party bargaining. The underlying model presented here has used the theory based on the existence of core parties and on the heart as an indication of the bargaining domain when the core is empty. Some countries, such as Israel at certain times, are characterized by the existence of a dominant party, able to attain enough seats to be strongly dominant and command the core position.

The main theoretical point that emerges is that there is hardly any centripetal tendency towards an electoral center. It is consistent with this analysis that activist groups will tend to pull the parties away from the center. Indeed, we can follow Duverger (1954) and Riker (1953) and note that under proportional electoral methods, there is very little motivation for interest groups to

coalesce. Consequently, the fragmentation of interest groups will lead to a degree of fragmentation in the polity. Fragmentation may be mitigated by the electoral system (especially if there is a relatively high electoral requirement which determines whether a party will obtain some legislative representation. However, even when there is a degree of majoritarianism in the electoral system this may have little effect on reducing fragmentation. For democratic polities, there may be an element of a quandary associated with the choice of an electoral system. If it is based on proportional representation then there may be the possibility of dominance by a centrally located party. Alternatively, there may be coalitional instability resulting from a fragmented polity and a complex configuration of parties.

The model with activist valence has also been briefly applied to the case of an autocrat, obtaining resources from various factor elites, in order to maintain power. It is implicit in the stochastic model that the outcome is non-deterministic. Although autocrat and counter-regime leaders may attempt to maximize some electoral operator, based on support, these operators can be expected to display considerable variance. The result of the contest between the autocrat and opponent should therefore be interpreted in probabilistic terms. As Przeworski et al. (2000) suggest, this may result in oscillation between authoritarian and democratic regimes.

7 References

Acemoglu, D,Robinson, J (2005). Economic origins of dictatorship and democracy. Cambridge University Press, Cambridge.

Adams, J (1999a). Multiparty spatial competition with probabilistic voting. Public Choice 99:259–274.

Adams, J (1999b). Policy divergence in multicandidate probabilistic spatial voting. Public Choice 100:103–122.

Adams, J (2001). Party competition and responsible party government. University of Michigan Press, Ann Arbor:

Adams, J Merrill III S (1999). Modelling party strategies and policy representation in multiparty elections: why are strategies so extreme? Am Jour Pol Sci 43:765–781.

Aldrich, J (1983a). A spatial model with party activists: implications for electoral dynamics. Public Choice 41: 63–100.

Aldrich, J (1983b). A Downsian spatial model with party activists. Am Pol Sci Rev 77:974–990.

Aldrich, J (1995). Why parties? Chicago University Press,Chicago

Arian A, Shamir M (1990). The election in Israel: 1988. SUNY Press, Albany.

Arian A, Shamir M (1995). The election in Israel: 1992. SUNY Press, Albany.

Arian A, Shamir M (1999). The election in Israel: 1996. SUNY Press, Albany.

Austen-Smith D, Banks J S (1999). positive political theory I: collective preferences. The University of Michigan Press, Ann Arbor:.

Banks, J, Duggan J (2000). A bargaining model of collective choice. Am Pol Sci Rev 94:73–88.
Banks, J, Duggan J (2005). The theory of probabilistic voting in the spatial model of elections. In Austen-Smith, D, Duggan, J. (eds),. Social Choice and Strategic Decisions. Springer, Berlin Heidelberg New York.
Banks, J, Duggan J, Le Breton M (2002). Bounds for mixed strategy equilibria and the spatial model of elections. Journ Econ Th 103: 88–105.
Banks, J, Duggan J, Le Breton M (2006) Social choice and electoral competition in the general spatial model. Journ Econ Th 126:194–234
Baron D P, Ferejohn, J A (1989) Bargaining in legislatures. Am Pol Sci Rev 83:1181–1206
Benoit, K, Laver, M. (2006). Party policy in modern democracies. Routledge, London.
Budge I, Klingemann H -D, Volkens A, Bara J (eds) (2001). Mapping policy preferences-estimates for parties, electors, and governments 1945–1998. Oxford University Press, Oxford
Budge I, Robertson D, Hearl D (eds)(1987) Ideology, strategy and party change: A spatial analysis of post-war election programmes in nineteen democracies. Cambridge University Press, Cambridge.
Bueno de Mesquita B, Morrow J D , Siverson R, Smith A (2003). The logic of political survival. MIT Press, Cambridge MA.
Cox G (1987).The uncovered set and the core. Am Jour Pol Sci 31:408–422.
Dow J K, Endersby, J (2004). Multinomial probit and multinomial logit: A comparison of choice models for voting research. Elect Stud 23:107-122.
Duverger M (1954). Political parties: Their organization and activity in the modern state. Wiley, New York.
Giannetti D, Sened I (2004). Party competition and coalition formation: Italy 1994-1996. Jour Theor Pol. 16:483–515.
Hinich M (1977). Equilibrium in spatial voting: The median voter result is an artifact. Jour Econ Th 16: 208-219.
Hotelling, H (1929). Stability in competition. Economic Journal 39:41–57.
Kramer G H (1978). Existence of electoral equilibrium. In Ordeshook,P.C. (ed.). Game theory and political science. New York University Press, New York.
Laver M, Budge I [eds]. 1992. Party policy and government coalitions. Macmillan,. London..
Laver M, Garry J (2000) Estimating policy positions from political texts. Am Pol Sci Rev 44:619-634.
Laver M, Hunt, W B (1992). Policy and party competition. Routledge, New York.
Laver M, Schofield N (1990). Multiparty government: The politics of coalition in Europe.Oxford University Press, Oxford. Reprinted 1998 University of Michigan Press, Ann Arbor.
Lin, T, Enelow, M J, Dorussen, H (1999). Equilibrium in multicandidate probabilistic spatial voting. Pub Choice 98:59–82.
McKelvey R D (1986). Covering, dominance and institution-free properties of social choice. Am Jour Pol Sci 30:283-314.

McKelvey R D, Patty J. W (2006). A theory of voting in large elections. Games Econ Behav 57:155–180.
McKelvey R D, Schofield N. (1987). Generalized symmetry conditions at a core point. Econometrica. 55:923–933.
North D. C, Weingast, B R (1989). Constitutions and commitment: the evolution of institutions governing public choice in seventeenth century England. Jour Econ Hist 49: 803–32.
Penn E (2003). A model of far-sighted voting. Typescript: Harvard University.
Penn E (2006) The Banks set in infinite spaces. Soc Choice Welf 27:531-543
Przeworski A, Alvarez, M E ,Cheibub J A , Limongi F (2000). Democracy and development: political institutions and well-being in the world, 1950–1990. Cambridge University Press, Cambridge.
Quinn K , Martin A, Whitford A (1999). Voter choice in multiparty democracies. Am Jour Pol Sci 43: 1231–1247
Riker W H (1953). Democracy in the United States.Macmillan, New York.
Riker W H, Ordeshook, P C (1973). An introduction to positive political theory. Prentice-Hall, Englewood Cliffs, NJ.
Schofield N (1985). Social choice and democracy. Springer, Berlin Heidelberg New York.
Schofield N. (1999). The heart and the uncovered set. Jour Econ, Suppl. 8,79–113.
Schofield N (2006a). Equilibria in the spatial stochastic model with party activists. The Rev Econ Design, 10: 183-203.
Schofield N (2006b). Architects of political change: constitutional quandaries and social choice theory. Cambridge University Press, Cambridge.
Schofield N (2007a) Political equilibrium with electoral uncertainty. Soc Choice Welf 28:461–490.
Schofield N (2007b) The mean voter theorem: necessary and sufficient conditions for convergent equilibrium. Rev Econ Stud 74:965-980.
Schofield N (2008). The spatial model of politics. Routledge, London
Schofield, N, Cataife G (2007). A model of political competition with activists with an application to the elections of 1989 and 1995 in Argentina. Math Soc Sci 53: 213-231.
Schofield N, Levinson M.. (2008). The collapse of authoritarian regimes. Pol Phil Econ:in press.
Schofield, N, Miller G, Martin A (2003). Critical elections and political realignments in the U.S.: 1860-2000. Pol Stud 51: 217-240.
Schofield N, Ozdemir U (2008) A spatial model of party positioning.Pub Choice:in press.
Schofield N, Sened I (2006) Multiparty democracy: elections and legislative Politics. Cambridge University Press, Cambridge.
Stokes D (1963) Spatial models and party competition Am Pol Sci Rev 57: 368–377.
Stokes D (1992). Valence politics. In: Kavanagh D (ed) Electoral politics. Clarendon Press, Oxford.

Table 1. Knesset Seats

Party	1988	1992	1996	1999	2003
Labor (LAB)	39	44	34	28	19[a]
Democrat (ADL)	1	2	4	5	2[a]
Meretz (MRZ)	–	12	9	10	6
CRM, MPM, PLP	9	–	–	–	3
Communist (HS)	4	3	5	3	3
Balad	–	–	–	2	3
Left Subtotal	*53*	*61*	*52*	*28*	*36*
Olim	–	–	7	6	2[b]
III Way	–	–	4	–	–
Center	–	–	–	6	–
Shinui (S)	2	–	–	6	15
Center Subtotal	*2*	–	*11*	*18*	*17*[b]
Likud (LIK)	40	32	30	19	38[b]
Gesher	–	–	2	–	–
Tzomet (TZ)	2	8	–	–	–
Israel Beiteinu	–	–	–	4	7
Subtotal	*42*	*40*	*32*	*23*	*45*
Shas (SHAS)	6	6	10	17	11
Yahadut (AI, DH)	7	4	4	5	5
Mafdal (NRP)	5	6	9	5	6
Moledet (MO)	2	3	2	4	–
Techiya (TY)	.3	–	–	–	–
Right Subtotal	*23*	*19*	*25*	*31*	*22*
TOTAL	120	120	120	120	120

[a] ADL, under Peretz, combined with Labor, to give 21 seats.
[b] Olim joined Likud to give 40 seats, and the right 47 seats.

Table 2 Turkish election results 1999

Party Name	.	% Vote	Seats	% Seats
Democratic Left Party	DSP	22.19	136	25
Nationalist Action Party	MHP	17.98	129	23
Virtue Party	FP	15.41	111	20
Motherland Party	ANAP	13.22	86	16
True Path Party	DYP	12.01	85	15
Republican People's Party	CHP	8.71	-	-
People's Democracy Party	HADEP	4.75	-	-
Others	-	4.86	-	-
Independents	-	0.87	3	1
Total			550	
Convergence Coefficient=2.014				

Table 3 Turkish election results 2002

Party Name		% Vote	Seats	% Seats
Justice and Development Party	AKP	34.28	363	66
Republican People's Party	CHP	19.39	178	32
True Path Party	DYP	9.54	-	-
Nationalist Action Party	MHP	8.36	-	-
Young Party	GP	7.25	-	-
People's Democracy Party	DEHAP	6.22	-	-
Motherland Party	ANAP	5.13	-	-
Felicity Party	SP	2.49	-	-
Democratic Left Party	DSP	1.22	-	-
Others and Independents	-	6.12	9	2
Total	-		550	
Convergence coefficient = 6.48				

Table 4 Turkish election results 2007

Party Name		% Vote	Seats	% Seats
Justice and Development Party	AKP	46.6	340	61.8
Republican People's Party	CHP	20.9	112	20.3
Nationalist Movement Party	MHP	14.3	71	12.9
Democrat Party	DP	5.4	-	-
Young Party	GP	3.0	-	-
Felicity Party	SP	2.3	-	-
Independents	-	5.2	27[20]	4.9
Others	-	2.3	-	-
Total		100	550	100

[20] Twenty-four of these "independents" were in fact members of the DTP–the Kurdish Freedom and Solidarity Party.

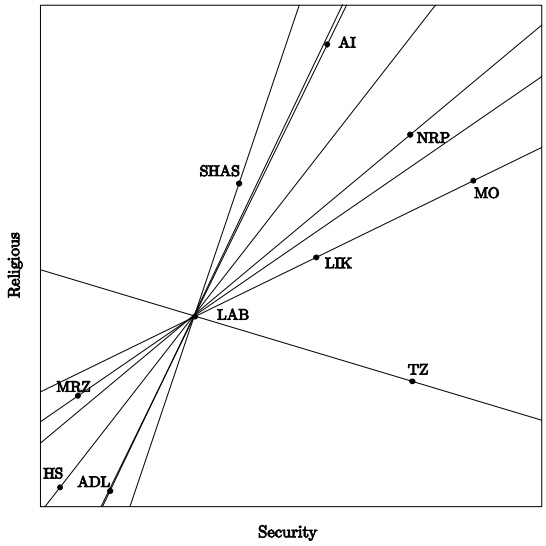

Figure 1: The core in the Knesset in 1992

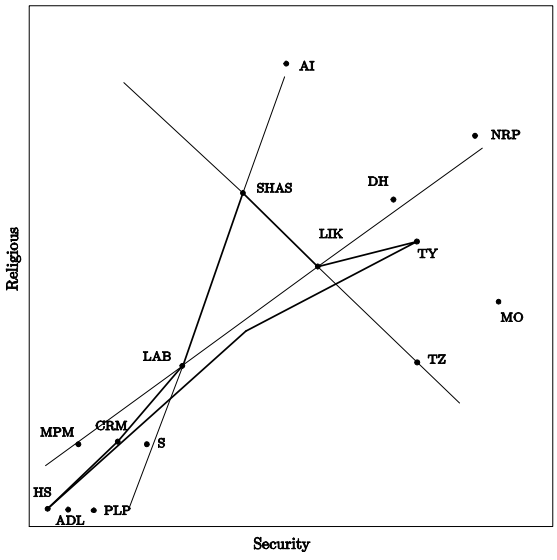

Figure 2: The heart in the Knesset in 1988

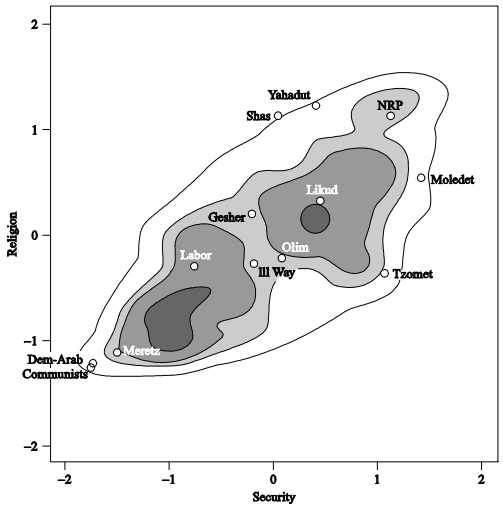

Figure 3: The Knesset in 1996

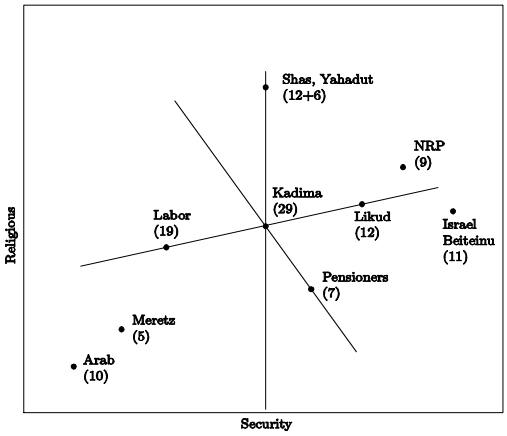

Figure 4: The Knesset in 2006

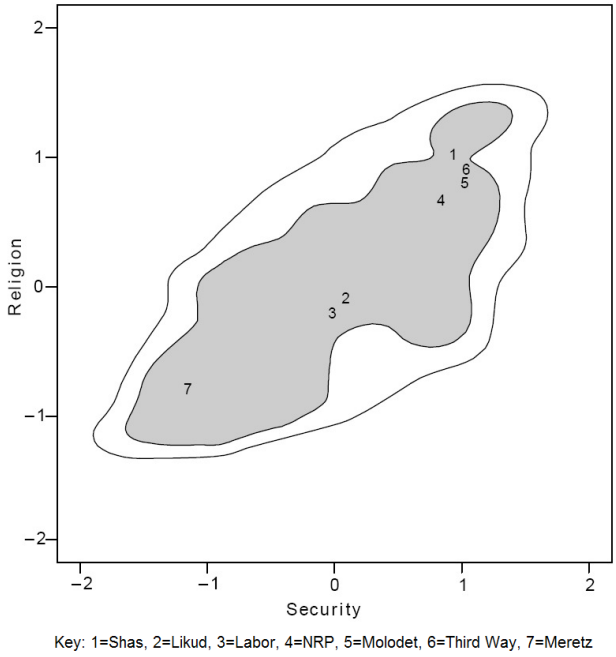

Figure 5: Simulated local Nash equilibrium in 1996 in the Knesset

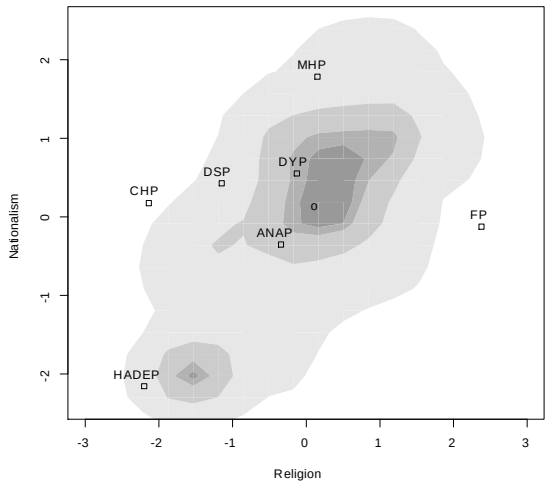

Figure 6: Party positions and voter distribution in Turkey in 1999

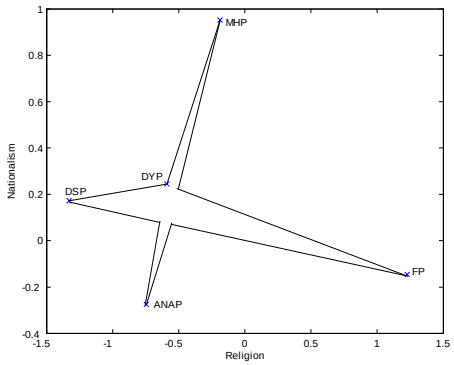

Figure 7: The heart in Turkey in 1999

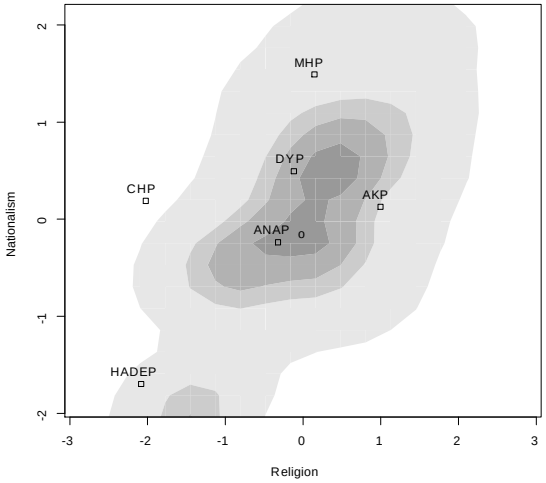

Figure 8: Party positions and voter distribution in Turkey in 2002

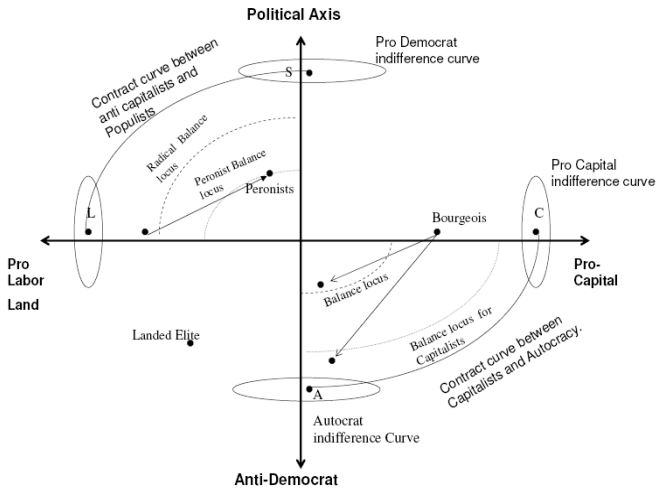

Figure 9: The contract curve for the capitalist/autocrat bargain

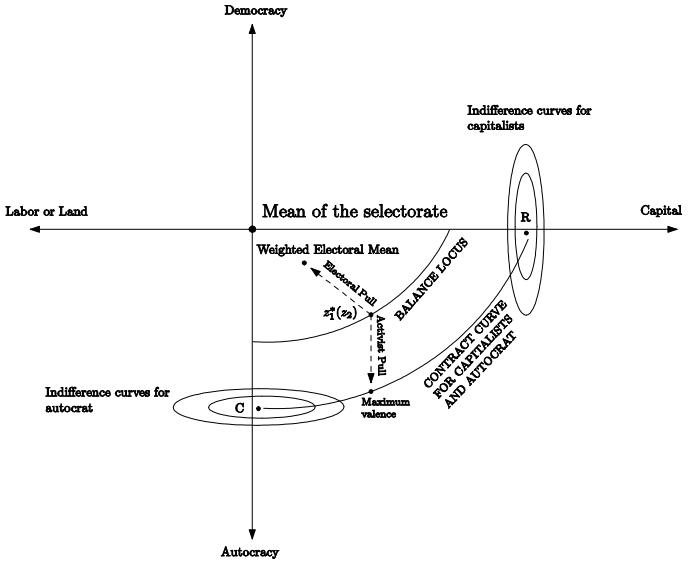

Figure 10: Optimum autocrat position